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ABSTRACT

This document is divided into two sections: general statistics on education in the United States and descriptions of changes in mathematics education in the U.S. Many tables and charts describe the structure of United States education, enrollment trends, expenditure patterns and revenue receipts, international enrollment, and staffing and classroom space. The section on mathematics education describes changes in mathematics curriculum and evaluation procedures that have taken place at elementary, secondary, and college levels. Mathematics equipment and learning aids, teacher education, and mathematics research are also discussed. Descriptions of federal and professional organizations that support programs for improving mathematics education conclude the presentation. A related document is ED 038 523. (Photographs may reproduce poorly.) (DN)

OE-10005-66-A

PROGRESS OF PUBLIC EDUCATION IN THE UNITED STATES OF AMERICA 1965-1966

REPORT OF THE OFFICE OF EDUCATION, U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE TO THE TWENTY-NINTH INTERNATIONAL CONFERENCE ON PUBLIC EDUCATION, GENEVA, SWITZERLAND JUNE 27–JULY 9, 1966, JOINTLY SPONSORED BY THE UNITED NATIONS EDUCATIONAL, SCIENTIFIC, AND CULTURAL ORGANIZATION AND THE INTERNATIONAL BUREAU OF EDUCATION

U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
John W. Gardner / *Secretary*
Office of Education / Harold Howe II / *Commissioner*

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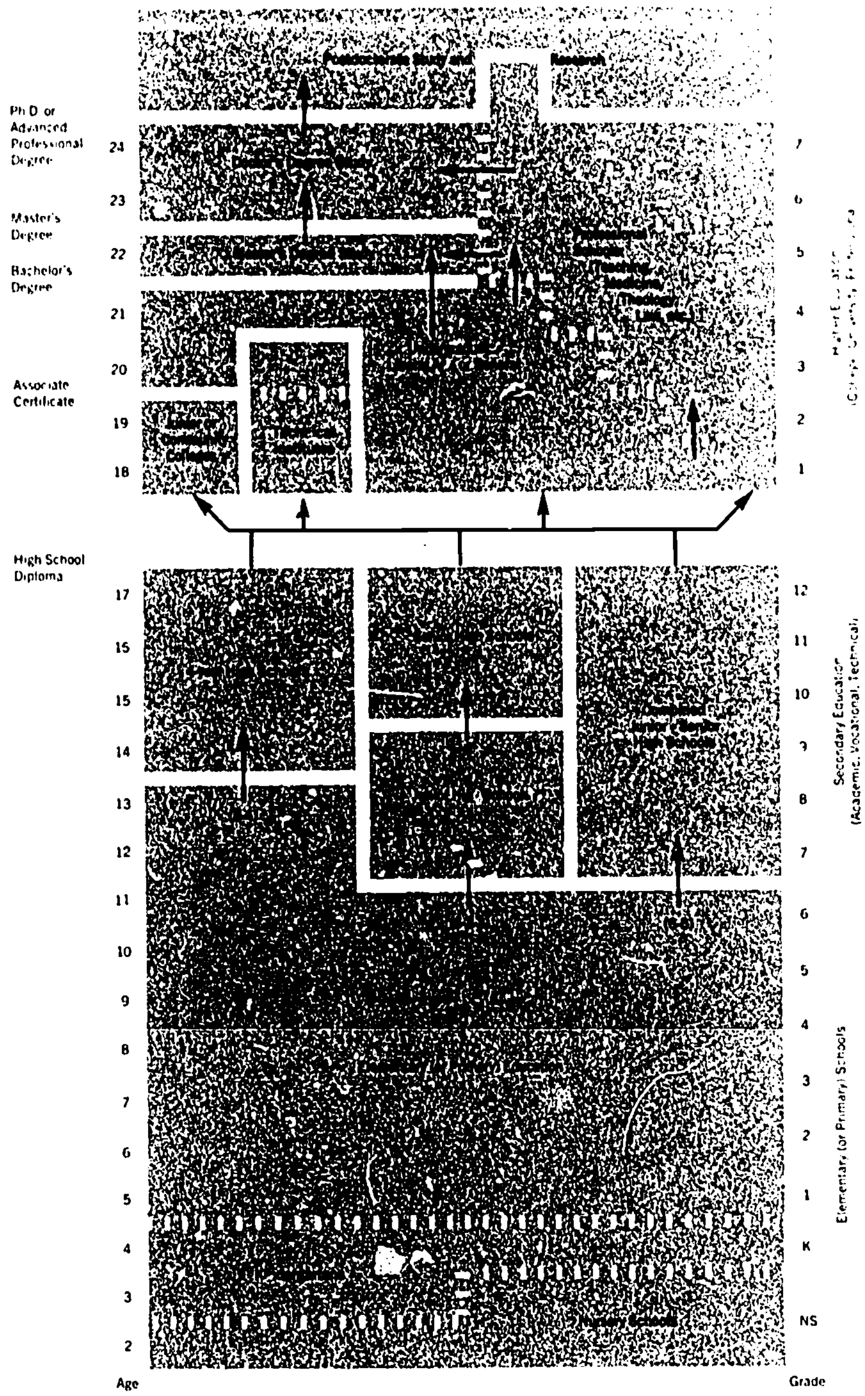
CONTENTS

PART ONE/GENERAL STATISTICS ON EDUCATION IN THE U.S.A.

PART TWO/MATHEMATICS EDUCATION IN THE U.S.A.

TABLES

FIGURES



THE STRUCTURE OF EDUCATION IN THE UNITED STATES

PART ONE/GENERAL STATISTICS ON EDUCATION IN THE U.S.A.

An Overview

The year 1965 was a banner period for education in the United States. More persons were enrolled in school than at any previous time. More teachers were employed in public and nonpublic schools than at any earlier date. The number of high school and college graduates reached an all-time high. The educational attainment of the population had never been higher, and illiteracy was rapidly being obliterated. More money, both in actual dollars and as a percentage of the gross national product, was spent on education than in any preceding year. The Federal Government, as well as the State and local governments, made substantial increases in the amount of support given to education. The paragraphs which follow present more detailed information on the progress of education in the United States.

Enrollment

In the fall of 1965, enrollment in educational institutions in the United States increased for the 21st consecutive year and reached another all-time high. The number of students in public and nonpublic institutions at all educational levels totaled 54.8 million (table 1). This total was 2.6 percent higher than the 53.4 million students enrolled 1 year earlier. The largest increase over the preceding year (11.6 percent) occurred at the higher education level. Enrollment in grades 9 through 12 rose 1.8 percent, while that in kindergarten through grade 8 increased 1.7 percent.

A major trend in American education during the 20th century has been the increasing number and proportion of students who enroll in secondary education programs. Table 2 provides some indication of the growth of secondary education in this country.

From 1890 to 1965, while the population 14 to 17 years of age rose little more than 2½ times, enrollment in grades 9 through 12 increased 36 times. In 1890 only about 1 person out of 15 in the 14–17 age group was enrolled in school; in 1965 the figure was more than 9 out of 10.

For nearly 50 years the Federal Government has helped State and local governments provide vocational education programs. In recent years new programs have been added to the traditional classes in agriculture, home economics, and trades and industry, and the number of participants has increased at a rapid rate. Almost 4.6 million students were enrolled in federally aided vocational classes in 1964 (table 3).

Teachers and Instructional Staff

As enrollment increases in the United States, there is a demand for more and more teachers at all educational levels. Between the fall of 1964 and 1965, the total teaching staff increased from 2.3 to 2.4 million, a rise of 4.4 percent (table 4).

In recent years the number of public elementary and secondary school teachers has risen at a faster rate than the number of pupils enrolled. Consequently, there has been a slight decline in the number of pupils per teacher. As table 5 indicates, there were 24.6 pupils per teacher in 1965 as compared with 25.8 pupils per teacher 5 years earlier.

Graduates

Paralleling the increase in school enrollment is a corresponding rise in the number and proportion of high school and college graduates. As recently as 1890, only 3.5 percent of our young people were graduating from high school. That year may be compared with the year 1965, when there were 2.7 million graduates, a number equal to 73 percent of the 17-year-olds in the population (table 6).

At the college level the contrast is even greater: The number of bachelor's degrees in 1965 was more than 33 times as great as in 1890, and the number of master's and doctor's degrees both increased more than 100 times (table 7).

School Retention Rates and Educational Attainment

Table 8 shows the increase in school retention rates from the fifth grade through college entrance over the past third of a century. During this period the proportion of fifth graders who go on to graduate from high school has increased 135 percent: About 71 percent of our former fifth graders graduated from high school in 1965, as compared with 30 percent in 1932. The increase in college attendance is even more striking: Approximately 38 percent of our young people now enter college; a generation ago the comparable figure was 12 percent. Retention rates for the high school graduating class of 1965 are shown graphically in figure 1.

Since 1940 the U.S. Bureau of the Census has collected statistics on the educational attainment of the population in this country. Table 9, which is derived from Census publications, compares the educational attainment of the population 25–29 years of age with the total population 25 years of age and over. The former group in March 1964 had completed nearly one more year of school than had the total adult population. More than two-thirds of the 25–29 age group were high school graduates, as compared with fewer than one-half of all adults. One-eighth of the 25–29-year-olds were college graduates, while only about 1 person in 11 among the total population had completed his college education.

Only 2.4 percent of the persons 14 years of age and over were illiterate in 1960 (table 10). This illiteracy rate may be compared with that of 3.3 percent in 1950, 4.8 percent in 1930, and 11.3 percent in 1900. Thus the 20th century has seen a steady reduction in the percentage of persons in this country who are unable to read and write.

Income

Public elementary and secondary schools in the United States derive most of their revenue from governmental sources. Income from other sources, such as gifts and fees, amounts to only about one-half of 1 percent of the total revenue receipts. Local governments contribute more than any other source, but in recent years an increasingly large proportion has come from State governments. In the school year 1963–64, the most recent year for which actual data are available, approximately 56 percent of the revenue

217-514 O - 66 - 2

receipts of public schools came from local sources, 40 percent from State governments, and 4 percent from the Federal Government (table 11, figure 2).

Although State and local governments have the primary responsibility for public education in the United States, the Federal Government for many years has maintained an active interest in the educational process. Recently an increasing amount of Federal support for all levels of education has been provided through a variety of programs administered by a number of Government agencies. Federal grants supporting education in educational institutions, for example, rose 70 percent between the fiscal years 1964 and 1965. Table 12 summarizes Federal grants, loans, and other funds for education and related activities for the past 2 years.

Expenditures

Estimated expenditures for public elementary and secondary education in the United States amounted to $23.1 billion during the school year 1964–65 and to $25.8 billion in 1965–66 (table 13). The estimated total expenditure per pupil in average daily attendance rose from $587 in 1964–65 to $641 in 1965–66. These figures may be compared with an expenditure of $388 only 10 years ago.

According to the latest available figures on expenditures by purpose, public schools are expending approximately 55 percent of their funds for instruction and 14 percent for capital outlay (figure 3). The remaining 31 percent is spent for a variety of purposes, including administration, plant operation and maintenance, fixed charges, other school services, and interest on school debt.

Table 14 compares total expenditures for all levels of public and private education in the United States with the gross national product over the past 35 years. Educational expenditures totaled approximately $39 billion during the school year 1964–65, an amount equal to about 6.2 percent of the gross national product. Preliminary estimates indicate that educational expenditures may have exceeded $41 billion in 1965–66. In terms of the gross national product, expenditures today are more than three times as great as they were during the middle 1940's.

Expenditures for vocational education from Federal, State, and local funds are shown in table 15. In 1963–64, the Federal Gov-

ernment contributed 17 percent of the funds, the State governments 38 percent, and the local governments 46 percent. Total expenditures for vocational education have more than doubled in the past decade.

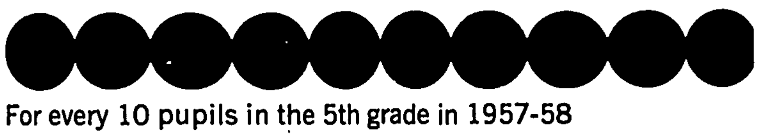
International Education

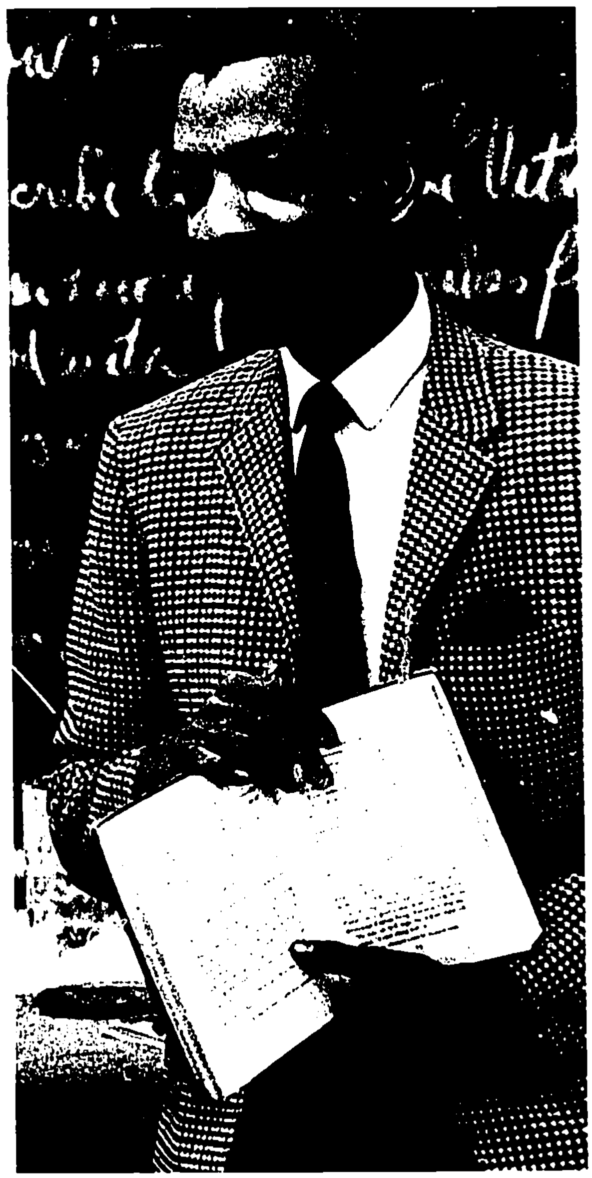
Table 16 shows the number of participants in international education programs administered by the Office of Education—the Teacher Development Program, the Teacher Exchange Program, and the Technical Assistance Program. Programs of the Federal Government represent only a fraction of the activities of the United States in the field of international education. This field embraces not only the students and teachers from abroad who come here each year, but also thousands of persons from the United States who go overseas each year to study, teach, and do research.

The number of students from abroad enrolled in institutions of higher education in the United States rose from 48,000 in 1959–60 to 82,000 in 1964–65, an increase of more than two-thirds (table 17). The fields in which the largest number of these students were enrolled in 1964–65 were engineering, humanities, physical and natural sciences, and social sciences. In the same year 36 percent of the overseas students came from the Far East, 17 percent from Latin America, 14 percent from the Near East, 12 percent from Europe, and the remaining 21 percent from various other parts of the world.

Definitive figures on the number of Americans studying overseas are not available. However, the Institute of International Education in New York has reported that at least 18,000 Americans were attending institutions of higher education abroad in 1963–64. The Institute has also reported that almost 9,000 faculty members, researchers, and scholars from overseas were at institutions of higher education in the United States in 1964–65 and that nearly 4,000 American faculty members were abroad in the same year.

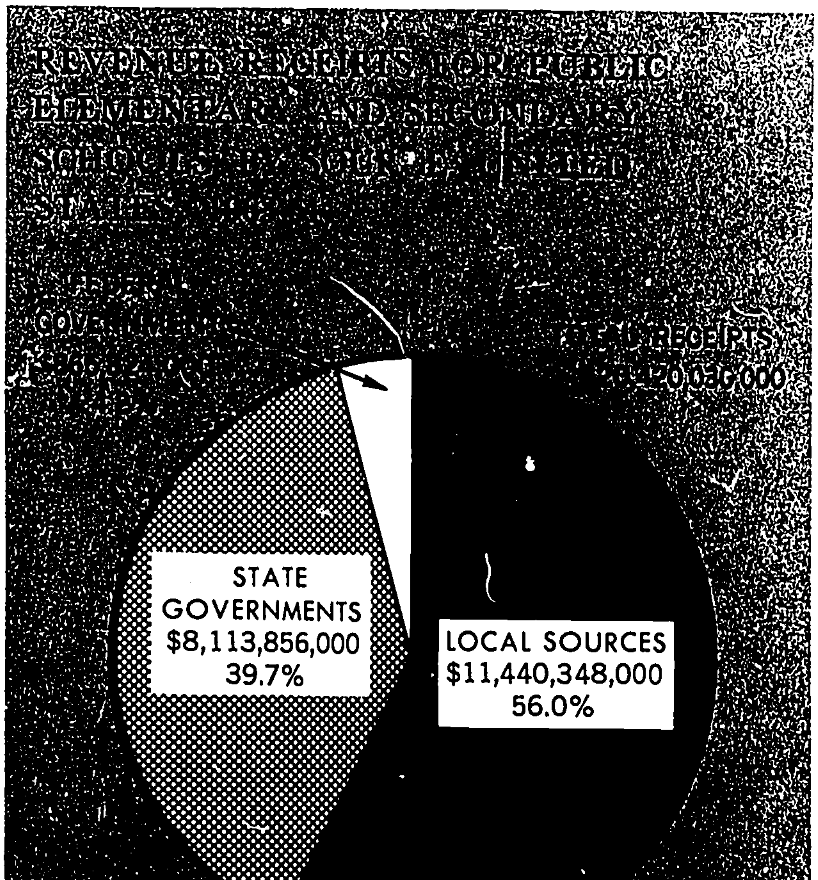
ESTIMATED SCHOOL RETENTION RATES: FIFTH GRADE THROUGH COLLEGE GRADUATION

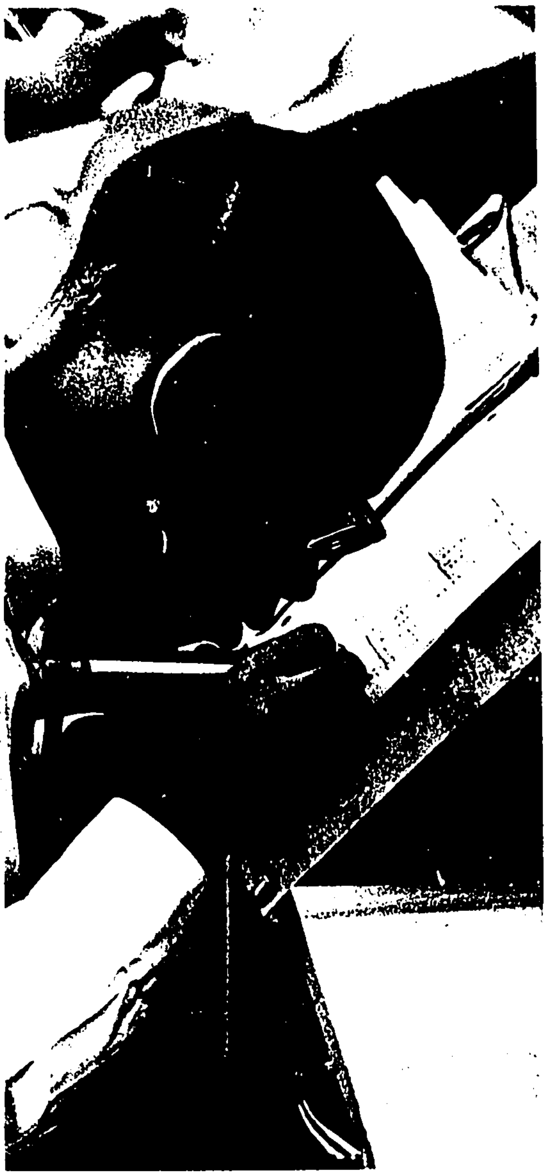




NOTE.—Compulsory attendance laws keep virtually all children in school at least until the fifth grade.

SOURCE: U.S. Department of Health, Education, and Welfare, Office of Education, Digest of Educational Statistics.


STATE GOVERNMENTS
$8,113,856,000
39.7%
LOCAL SOURCES
$11,440,348,000
56.0%



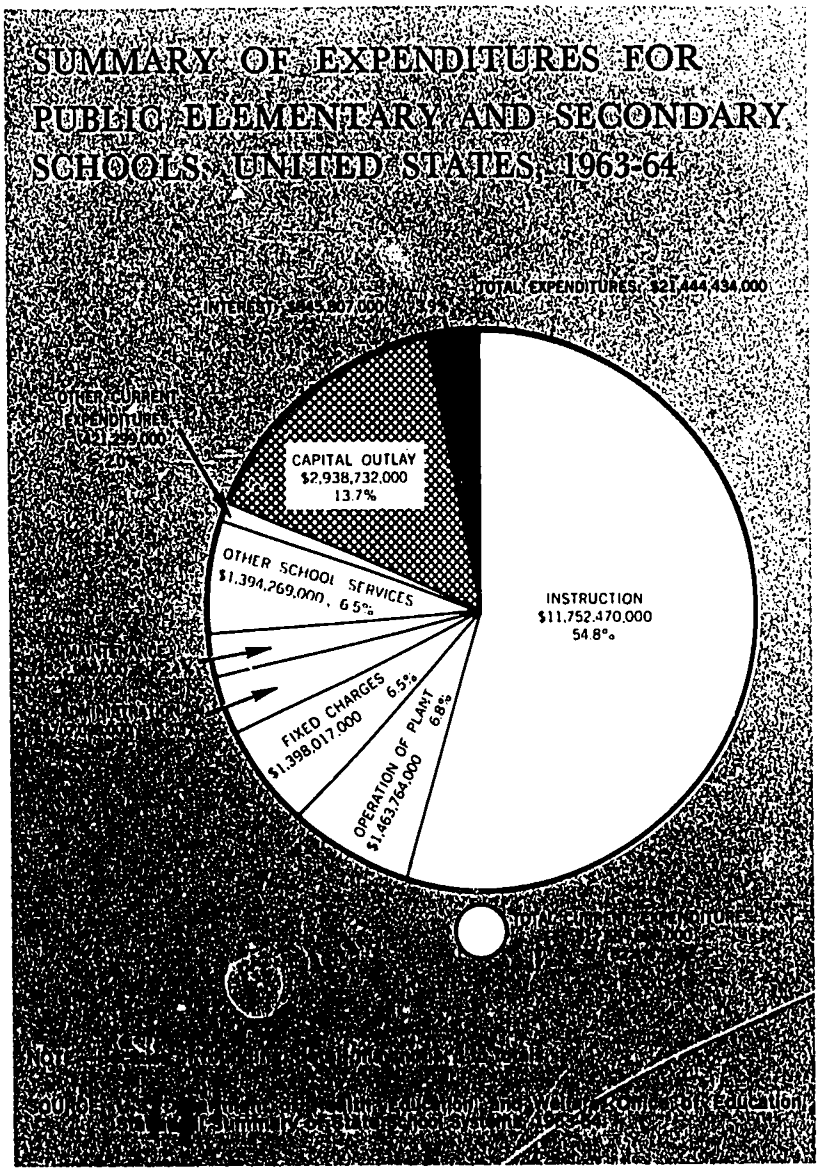

SUMMARY OF EXPENDITURES FOR PUBLIC ELEMENTARY AND SECONDARY SCHOOLS, UNITED STATES, 1963-64
TOTAL EXPENDITURES: $21,444,434,000
CAPITAL OUTLAY $2,938,732,000 13.7%
OTHER SCHOOL SERVICES $1,394,269,000, 6.5%
INSTRUCTION $11,752,470,000 54.8%
FIXED CHARGES $1,398,017,000 6.5%
OPERATION OF PLANT $1,463,764,000 6.8%

Table 1 Fall enrolment in educational institutions, by grade level and type of school: United States, fall 1964 and 1965

Grade level and type of school	Fall 1964	Fall 1965	Percentage increase, 1964 to 1965
Total, elementary, secondary, and higher education	**53,366,000**	**54,770,000**	**2.6**
Kindergarten through grade 8	35,525,000	36,126,000	1.7
Public school systems (regular full-time)	30,025,000	30,526,000	1.7
Nonpublic schools (regular full-time)[1]	5,300,000	5,400,000	1.9
Other schools[1 2]	200,000	200,000	.0
Grades 9 through 12	12,891,000	13,118,000	1.8
Public school systems (regular full-time)	11,391,000	11,618,000	2.0
Nonpublic schools (regular full-time)[1]	1,400,000	1,400,000	.0
Other schools[1 2]	100,000	100,000	.0
Kindergarten through grade 12	48,416,000	49,244,000	1.7
Public school systems (regular full-time)	41,416,000	42,144,000	1.8
Nonpublic schools (regular full-time)[1]	6,700,000	6,800,000	1.5
Other schools[1 2]	300,000	300,000	.0
Higher education: Universities, colleges, professional schools, junior colleges, normal schools, and teachers colleges (degree-credit enrollment)	4,950,000	5,526,000	11.6

[1] Estimated.
[2] Includes federally operated schools, subcollegiate departments of institutions of higher education, and residential schools for exceptional children.

NOTE.—Fall enrollment is usually smaller than school-year enrollment, since the latter is a cumulative figure which includes students who enroll at any time during the year.

Source: U.S. Department of Health, Education, and Welfare, Office of Education, surveys and estimates of the National Center for Educational Statistics.

Table 2 Enrollment in grades 9–12 of public and nonpublic schools compared with population 14–17 years of age: United States, 1889–90 to fall 1965

School year	Enrollment, grades 9-12 and postgraduate [1]			Population 14-17 years of age [2]	Total number enrolled per 100 persons 14-17 years of age
	All schools	Public schools	Nonpublic schools		
1889-90	359, 949	[3] 202, 963	[3] 94, 931	5, 354, 653	6. 7
1899-1900	699, 403	[3] 519, 251	[3] 110, 797	6, 152, 231	11. 4
1909-10	1, 115, 398	[3] 915, 061	[3] 117, 400	7. 220, 298	15. 4
1919-20	2, 500, 176	[3] 2, 200, 389	[3] 213, 920	7, 735, 841	32. 3
1929-30	4, 804, 255	[3] 4, 399, 422	[3 4] 341, 158	9. 341, 221	51. 4
1939-40	7, 123, 009	6, 635, 337	487, 672	9, 720, 419	73. 3
1949-50	6, 453, 009	5, 757, 810	695, 199	8, 404, 768	76. 8
1951-52	6, 596, 351	5, 917, 384	678, 967	[5] 8, 516, 000	77. 5
1953-54	7, 108, 973	6, 330, 565	778, 408	[5] 8, 861, 000	80. 2
1955-56	7, 774, 975	6, 917, 790	857, 185	[5] 9, 207, 000	84. 4
1957-58	8, 869, 186	7, 905, 469	963, 717	[5] 10, 139, 000	87. 5
1959-60	9, 599, 810	8, 531, 454	1. 068, 356	11, 154, 879	86. 1
1961-62	10, 768, 972	9, 616, 755	1, 152, 217	[5] 12, 006, 000	89. 7
Fall 1963	12, 254, 000	10, 935, 000	1, 319, 000	[5] 13, 499, 000	90. 8
Fall 1965 [6]	13, 118, 000	11, 678, 000	1, 440, 000	[5] 14, 110, 000	93. 0

[1] Unless otherwise indicated, includes enrollment in subcollegiate departments of institutions of higher education and in residential schools for exceptional children. Beginning in 1949-50, also includes Federal schools.

[2] Includes all persons residing in the United States, but excludes Armed Forces overseas. Data shown are actual figures from the decennial censuses of population unless otherwise indicated.

[3] Excludes enrollment in subcollegiate departments of institutions of higher education and in residential schools for exceptional children.

[4] Data for 1927-28.

[5] Estimated by the Bureau of the Census as of July 1 preceding the opening of the school year.

[6] Preliminary data.

NOTE.—Beginning in 1959-60, includes Alaska and Hawaii.

Source: U.S. Department of Health, Education, and Welfare, Office of Education, "Digest of Educational Statistics."

Table 3 Enrollment in federally aided vocational classes, by type of program: United States and outlying areas, 1919–20 to 1963–64

School year	Total	Type of program					
		Agriculture	Distributive occupations	Home economics	Trades and industry	Health occupations	Technical education
1919-20	265, 058	31, 301	....	48, 938	184, 819	....	....
1929-30	981, 882	188, 311	....	174, 967	618, 604	....	....
1939-40	2, 290, 741	584, 133	129, 433	818, 766	758, 409	....	....
1949-50	3, 364, 613	764, 975	364, 670	1, 430, 366	804, 602	....	....
1951-52	3, 165, 988	746, 402	234, 984	1, 391, 389	793, 213	....	....
1953-54	3, 164, 851	737, 502	220, 619	1, 380, 147	826, 583	....	....
1955-56	3, 413, 159	785, 599	257, 025	1, 486, 816	883, 719	....	....
1957-58	3, 629, 339	775, 892	282, 558	1, 559, 822	983, 644	27, 423	....
1959-60	3, 768, 149	796, 237	303, 784	1, 588, 109	938, 490	40, 250	101, 279
1961-62	4, 072, 677	822, 664	321, 065	1, 725, 660	1, 005, 383	48, 985	148, 920
1963-64	4, 566, 390	860, 605	334, 126	2, 022, 138	1, 069, 274	59, 006	221, 241

Source: U.S. Department of Health, Education, and Welfare, Office of Education, "Digest of Annual Reports of State Boards for Vocational Education."

Table 4 Number of classroom teachers in elementary and secondary schools, and instructional staff members in institutions of higher education: United States, fall 1964 and 1965 [1]

[Includes full-time and part-time teachers and staff]

Level and type of school	Fall 1964	Fall 1965	Percentage increase, 1964 to 1965
All levels	**2,286,000**	**2,387,000**	**4.4**
Elementary schools	1,106,000	1,137,000	2.8
Public (regular full-time)	942,000	968,000	2.8
Nonpublic (regular full-time)	150,000	155,000	3.5
Other [2]	14,000	14,000	.0
Secondary schools	796,000	836,000	5.0
Public (regular full-time)	709,000	749,000	5.6
Nonpublic (regular full-time)	80,000	80,000	.0
Other [2]	7,000	7,000	.0
Elementary and secondary schools	1,902,000	1,972,000	3.7
Public (regular full-time)	1,651,000	1,716,000	3.9
Nonpublic (regular full-time)	230,000	235,000	2.2
Other [2]	21,000	21,000	.0
Higher education [3]	384,000	415,000	8.1
Public	212,000	229,000	8.0
Nonpublic	172,000	186,000	8.1

[1] All figures except those for public elementary and secondary schools are estimated.
[2] Includes federally operated schools, subcollegiate departments of institutions of higher education, and residential schools for exceptional children.
[3] Includes faculty for resident instruction in degree-credit courses; excludes faculty engaged in administration, research, extension work, etc.

NOTE.—Because of rounding, detail may not add to totals.

Source: U.S. Department of Health, Education, and Welfare, Office of Education, surveys and estimates of the National Center for Educational Statistics.

217-514 O - 66 - 3

Table 5 Comparative statistics on enrollment, teachers, and schoolhousing in full-time public elementary and secondary schools: United States, fall 1960 and 1965

Item	Fall 1960	Fall 1965	Percentage change, 1960 to 1965
ENROLLMENT			
Total	**36, 281, 294**	**42, 143, 504**	**+16. 2**
Elementary schools	24, 349, 932	26, 415, 834	+8. 5
Secondary schools	11, 931, 362	15, 727, 670	+31. 8
CLASSROOM TEACHERS			
Total	**1, 408, 093**	**1, 716, 285**	**+21. 9**
Elementary schools	858, 249	967, 635	+12. 7
Secondary schools	549, 844	748, 650	+36. 2
PUPIL-TEACHER RATIO			
All schools	**25. 8**	**24. 6**	....
Elementary schools	28. 4	27. 3	....
Secondary schools	21. 7	21. 0	....
INSTRUCTION ROOMS			
Total available	**1, 331, 624**	**1, 595, 150**	**+19. 8**
Number completed during preceding school year	69, 400	65, 200	−6. 1

Source: U.S. Department of Health, Education, and Welfare, Office of Education, "Fall 1960 Enrollment, Teachers, and Schoolhousing" "and Fall 1965 Statistics of Public Schools."

Table 6 Number of high school graduates compared with population 17 years of age: United States, 1869-70 to 1964-65

School year	Population 17 years old [2]	High school graduates [1]			Number graduated per 100 persons 17 years of age
		Total	Boys	Girls	
1869-70	815, 000	16, 000	7, 064	8, 936	2. 0
1879-80	946, 026	23, 634	10, 605	13, 029	2. 5
1889-90	1, 259, 177	43, 731	18, 549	25, 182	3. 5
1899-1900	1, 489, 146	94, 883	38, 075	56, 808	6. 4
1909-10	1, 786, 240	156, 429	63, 676	92, 753	8. 8
1919-20	1, 855, 173	311, 266	123, 684	187, 582	16. 8
1929-30	2, 295, 822	666, 904	300, 376	366, 528	29. 0
1939-40	2, 403, 074	1, 221, 475	578, 718	642, 757	50. 8
1949-50	2, 034, 450	1, 199, 700	570, 700	629, 000	59. 0
1951-52	2, 040, 800	1, 196, 500	569, 200	627, 300	58. 6
1953-54	2, 128, 600	1, 276, 100	612, 500	663, 600	60. 0
1955-56	2, 270, 000	1, 414, 800	679, 500	735, 300	62. 3
1957-58	2, 324, 000	1, 505, 900	725, 500	780, 400	64. 8
1959-60	2, 862, 005	1, 864, 000	898, 000	966, 000	65. 1
1961-62	2, 768, 000	1, 925, 000	941, 000	984, 000	69. 5
1963-64	3, 001, 000	2, 290, 000	1, 121, 000	1, 169, 000	76. 3
1964-65 [3]	3, 670, 000	2, 668, 000	1, 315, 000	1, 353, 000	72. 7

[1] Includes graduates of public and nonpublic schools.
[2] Data from the Bureau of the Census.
[3] Preliminary data.

NOTE.—Beginning in 1959-60, includes Alaska and Hawaii.

Source: U.S. Department of Health, Education, and Welfare, Office of Education, "Digest of Educational Statistics."

Table 7 Earned degrees conferred by institutions of higher education: United States, 1869-70 to 1964-65

Year	Earned degrees conferred			
	All degrees	Bachelor's and first professional	Master's except first professional	Doctor's
1869-70	9, 372	9, 371	0	1
1879-80	13, 829	12, 896	879	54
1889-90	16, 703	15, 539	1, 015	149
1899-1900	29, 375	27, 410	1, 583	382
1909-10	39, 755	37, 199	2, 113	443
1919-20	53, 516	48, 622	4, 279	615
1929-30	139, 752	122, 484	14, 969	2, 299
1939-40	216, 521	186, 500	26, 731	3, 290
1949-50	496, 661	432, 058	58, 183	6, 420
1951-52	401, 203	329, 986	63, 534	7, 683
1953-54	356, 608	290, 825	56, 788	8, 995
1955-56	376, 973	308, 812	59, 258	8, 903
1957-58	436, 979	362, 554	65, 487	8, 938
1959-60	476, 704	392, 440	74, 435	9, 829
1961-62	514, 323	417, 846	84, 855	11, 622
1963-64	614, 194	498, 654	101, 050	14, 490
1964-65 [1]	651, 300	525, 000	111, 000	15, 300

[1] Estimated.

NOTE.—Beginning in 1959-60, includes Alaska and Hawaii.

Source: U.S. Department of Health, Education, and Welfare, Office of Education, "Digest of Educational Statistics" and circulars on "Earned Degrees Conferred."

Table 8 Estimated retention rates, 5th grade through college entrance, in public and nonpublic schools: United States, 1924–32 to 1957–65

School year in which pupils entered 5th grade	For every 1,000 pupils entering 5th grade in a specified year, this number— Entered 6th grade 1 year later	Entered 7th grade 2 years later	Entered 8th grade 3 years later	Entered 9th grade 4 years later	Entered 10th grade 5 years later
1924–25	911	798	741	612	470
1926–27	919	824	754	677	552
1928–29	939	847	805	736	624
1930–31	943	872	824	770	652
1932–33	935	889	831	786	664
1934–35	953	892	842	803	711
1936–37	954	895	849	839	704
1938–39	955	908	853	796	655
1940–41	968	910	836	781	697
1942–43	954	909	847	807	713
1944–45	952	929	858	848	748
1946–47	954	945	919	872	775
1948–49	984	956	929	863	795
1950–51	981	968	921	886	809
1952–53	974	965	936	904	835
1954–55	980	979	948	915	855
1956–57	985	984	948	930	871
1957–58	994	985	954	937	878

School year in which pupils entered 5th grade	Entered 11th grade 6 years later	Entered 12th grade 7 years later	Graduated from high school 7 years later (i.e., in the year shown)	Entered college 8 years later
1924–25	384	344	302 (in 1932)	118
1926–27	453	400	333 (in 1934)	129
1928–29	498	432	378 (in 1936)	137
1930–31	529	463	417 (in 1938)	148
1932–33	570	510	455 (in 1940)	160
1934–35	610	512	467 (in 1942)	129
1936–37	554	425	393 (in 1944)	121
1938–39	532	444	419 (in 1946)	(1)
1940–41	566	507	481 (in 1948)	(1)
1942–43	604	539	505 (in 1950)	205
1944–45	650	549	522 (in 1952)	234
1946–47	641	583	553 (in 1954)	283
1948–49	706	619	581 (in 1956)	301
1950–51	709	632	582 (in 1958)	308
1952–53	746	667	621 (in 1960)	328
1954–55	759	684	642 (in 1962)	343
1956–57	785	724	667 (in 1964)	357
1957–58	810	758	710 (in 1965)	378

[1] Lack of detailed information about students who were veterans prevents reliable calculation.

Source: U.S. Department of Health, Education, and Welfare, Office of Education, "Digest of Educational Statistics."

Table 9 **Level of school completed by persons 25 years old and over, and 25 to 29 years old: United States, 1940 to 1964**

Date and age	Percent by level of school completed			Median school years completed
	Fewer than 5 years of elementary school	4 years of high school or more	4 or more years of college	
25 years and over				
March 1964	7.1	48.0	9.1	11.7
March 1962	7.8	46.3	8.9	11.4
March 1959	8.0	42.9	7.9	11.0
March 1957	9.0	40.8	7.5	10.6
October 1952	9.1	38.4	6.9	10.1
April 1950	10.8	33.4	6.0	9.3
April 1947	10.4	32.6	5.4	9.0
April 1940	13.5	24.1	4.6	8.4
25 to 29 years				
March 1964	2.1	69.2	12.8	12.4
March 1962	2.4	65.9	13.1	12.4
March 1959	3.0	63.3	11.0	12.3
October 1952	3.8	56.7	10.0	12.2
April 1950	4.6	51.7	7.7	12.1
April 1940	5.9	37.8	5.8	10.4

NOTE.—Beginning in 1962, includes Alaska and Hawaii. Data for 1962 and 1964 are not strictly comparable with earlier years.

Source: U.S. Department of Commerce, Bureau of the Census, "Current Population Reports," Series P-20, Nos. 99, 121, and 138.

Table 10 **Percent of illiteracy [1] in the population: United States, 1900 to 1960**

Year	Percent illiterate [2]	Year	Percent illiterate [2]
1900	11.3	1930	4.8
1910	8.3	1950 [3]	3.3
1920	6.5	1960 [3]	2.4

[1] Illiteracy is defined as the inability to read and write a simple message either in English or in any other language.
[2] Percentages refer to the population 15 years old and over from 1900 to 1930 and to the population 14 years old and over in 1950 and 1960.
[3] Estimated.

NOTE.—Data are for 50 States and the District of Columbia.

Source: U.S. Department of Commerce, Bureau of the Census, "Current Population Reports," Series P-23, No. 8.

Table 11 **Revenue receipts for public elementary and secondary schools by source: United States, 1919–20 to 1964–65**

School year	Total	Federal Government	State governments	Local sources [1]
	AMOUNTS			
1919–20	$970, 120, 000	$2, 475, 000	$160, 085, 000	$807, 561, 000
1929–30	2, 088, 557, 000	7, 334, 000	353, 670, 000	1, 727, 553, 000
1939–40	2, 260, 527, 000	39, 810, 000	684, 354, 000	1, 536, 363, 000
1949–50	5, 437, 044, 000	155, 848, 000	2, 165, 689, 000	3, 115, 507, 000
1951–52	6, 423, 816, 000	227, 711, 000	2, 478, 596, 000	3, 717, 507, 000
1953–54	7, 866, 852, 000	355, 237, 000	2, 944, 103, 000	4, 567, 512, 000
1955–56	9, 686, 677, 000	441, 442, 000	3, 828, 886, 000	5, 416, 350, 000
1957–58	12, 181, 513, 000	486, 484, 000	4, 800, 368, 000	6, 894, 661, 000
1959–60	14, 746, 618, 000	651, 639, 000	5, 768, 047, 000	8, 326, 932, 000
1961–62	17, 527, 707, 000	760, 975, 000	6, 789, 190, 000	9, 977, 542, 000
1963–64	20, 420, 030, 000	865, 824, 000	8, 113, 856, 000	11, 440, 348, 000
1964–65 [2]	22, 083, 000, 000	936, 000, 000	8, 774, 000, 000	12, 373, 000, 000
	PERCENTAGE DISTRIBUTION			
1919–20	100. 0	0. 3	16. 5	83. 2
1929–30	100. 0	. 4	16. 9	82. 7
1939–40	100. 0	1. 8	30. 3	68. 0
1949–50	100. 0	2. 9	39. 8	57. 3
1951–52	100. 0	3. 5	38. 6	57. 9
1953–54	100. 0	4. 5	37. 4	58. 1
1955–56	100. 0	4. 6	39. 5	55. 9
1957–58	100. 0	4. 0	39. 4	56. 6
1959–60	100. 0	4. 4	39. 1	56. 5
1961–62	100. 0	4. 3	38. 7	56. 9
1963–64	100. 0	4. 2	39. 7	56. 0
1964–65 [2]	100. 0	4. 2	39. 7	56. 0

[1] Includes receipts from intermediate and local governments. Also includes a relatively small amount from other sources (gifts and tuition and transportation fees from patrons), which accounted for 0.6 percent of total revenue receipts in 1963-64.

[2] Total revenue receipts are estimated to be 15.63 percent higher than the expected total current expenditures (as in 1963-64). Distribution by source is estimated on the basis of the 1963-64 percentage distribution.

NOTE.—Beginning in 1959-60, includes Alaska and Hawaii. Because of rounding, detail may not add to totals.

Source of final data: U.S. Department of Health, Education, and Welfare, Office of Education, "Statistical Summary of State School Systems. 1963-64."

Table 12 Federal funds for education and related activities: Fiscal years 1964 and 1965

Level and type of support	1964	1965	Percentage change, 1964 to 1965
Federal funds supporting educational institutions: [1]			
Grants, total	**$2,251,100,000**	**$3,828,700,000**	**+70.1**
Elementary-secondary education	605,500,000	1,029,200,000	+70.0
Higher education	1,333,700,000	2,042,100,000	+53.1
Adult, vocational-technical, and continuing education	203,800,000	637,200,000	+212.7
Not classified by level	108,100,000	120,200,000	+11.2
Loans, total	**399,300,000**	**414,300,000**	**+3.8**
Elementary-secondary education	500,000	400,000	−20.0
Higher education	398,800,000	413,900,000	+3.8
Other Federal funds for education and related activities: [2]			
Applied research and development	1,052,900,000	1,104,200,000	+4.9
Related school services	421,400,000	509,500,000	+20.9
Training of Federal personnel	1,430,000,000	1,543,900,000	+8.0
Library services	25,500,000	76,500,000	+200.0
International education	93,800,000	93,800,000	.0
Other	96,800,000	103,400,000	+6.8

[1] Excludes payments for services rendered to the Federal Government.
[2] Includes payments for services rendered to the Federal Government.

Source: U.S. Department of Health, Education, and Welfare, Office of Education. "Digest of Educational Statistics."

Table 13 Total and per-pupil expenditures for public elementary and secondary education: United States, 1919-20 to 1965-66

School year	Total	Total expenditure per pupil in average daily attendance
1919-20	$1, 036, 151, 000	$64
1929-30	2, 316, 790, 000	108
1939-40	2, 344, 049, 000	106
1949-50	5, 837, 643, 000	259
1951-52	7, 344, 237, 000	313
1953-54	9, 092, 449, 000	351
1955-56	10, 955, 047, 000	388
1957-58	13, 569, 163, 000	449
1959-60	15, 613, 255, 000	472
1961-62	18, 373, 339, 000	518
1963-64	21, 444, 434, 000	562
1964-65 [1]	23, 106, 854, 000	587
1965-66 [1]	25, 801, 995, 000	641

[1] Estimated.

NOTE.—Beginning in 1959-60, includes Alaska and Hawaii.

Source: U.S. Department of Health, Education, and Welfare, Office of Education, "Statistics of State School Systems, 1963-64" and "Fall Statistics of Public Schools," 1964 and 1965.

Table 14 Gross national product related to total expenditures[1] for education: United States, 1929-30 to 1964-65

Calendar year	Gross national product	School year	Expenditures for education: Total	Expenditures for education: As a percent of gross national product
1929	$103, 095, 000, 000	1929-30	$3, 233, 601, 000	3. 14
1931	75, 820, 000, 000	1931-32	2, 966, 464, 000	3. 91
1933	55, 601, 000, 000	1933-34	2, 294, 896, 000	4. 13
1935	72, 247, 000, 000	1935-36	2, 649, 914, 000	3. 67
1937	90, 446, 000, 000	1937-38	3, 014, 074, 000	3. 33
1939	90, 494, 000, 000	1939-40	3, 199, 593, 000	3. 54
1941	124, 540, 000, 000	1941-42	3, 203, 548, 000	2. 57
1943	191, 592, 000, 000	1943-44	3, 522, 007, 000	1. 84
1945	212, 010, 000, 000	1945-46	4, 167, 597, 000	1. 97
1947	231, 323, 000, 000	1947-48	6, 574, 379, 000	2. 84
1949	256, 484, 000, 000	1949-50	8, 795, 635, 000	3. 43
1951	328, 404, 000, 000	1951-52	11, 312, 446, 000	3. 44
1953	364, 593, 000, 000	1953-54	13, 949, 876, 000	3. 83
1955	397, 960, 000, 000	1955-56	16, 811, 651, 000	4. 22
1957	441, 134, 000, 000	1957-58	21, 119, 565, 000	4. 79
1959	483, 650, 000, 000	1959-60	24, 722, 464, 000	5. 11
1961	520, 109, 000, 000	1961-62	29, 366, 305, 000	5. 65
1963	589, 238, 000, 000	1963-64	[2] 35, 900, 000, 000	6. 09
1964	628, 699, 000, 000	1964-65	[2] 39, 000, 000, 000	6. 20

[1] Includes expenditures of public and nonpublic schools at all levels of education (elementary, secondary, and higher education).

[2] Estimated.

NOTE.—Beginning with 1959-60 school year, includes Alaska and Hawaii.

Sources: U.S. Department of Health, Education, and Welfare, Office of Education, "Digest of Educational Statistics;" and U.S. Department of Commerce, Office of Business Economics, "Survey of Current Business," August 1965.

Table 15 Expenditure of Federal, State, and local funds for vocational education: United States and outlying areas, 1919-20 to 1963-64

School year	Total	Federal	State	Local
1919-20	$8, 535, 000	$2, 477, 000	$2, 670, 000	$3, 388, 000
1929-30.............	29, 909, 000	7, 404, 000	8, 233, 000	14, 272, 000
1939-40.............	55, 081, 000	20, 004, 000	11, 737, 000	23, 340, 000
1949-50.............	128, 717, 000	26, 623, 000	40, 534, 000	61, 561, 000
1951-52.............	146, 466, 000	25, 863, 000	47, 818, 000	72, 784, 000
1953-54.............	151, 289, 000	25, 419, 000	54, 550, 000	71, 320, 000
1955-56.............	175, 886, 000	33, 180, 000	61, 821, 000	80, 884, 000
1957-58.............	209, 748, 000	38, 733, 000	72, 305, 000	98, 710, 000
1959-60.............	238, 812, 000	45, 313, 000	82, 466, 000	111, 033, 000
1961-62.............	283, 948, 000	51, 438, 000	104, 264, 000	128, 246, 000
1963-64.............	332, 785, 000	55, 027, 000	124, 975, 000	152, 784, 000

NOTE.—Because of rounding, detail may not add to totals.

Source: U.S. Department of Health, Education, and Welfare, Office of Education, "Digest of Annual Reports of State Boards for Vocational Education."

Table 16 Number of participants in international education programs administered by the U.S. Office of Education: 1955-56, 1960-61, and 1965-66

Program	Number of participants		
	1955-56	1960-61	1965-66
Teacher development........................	265	529	[1] 582
Teacher exchange...........................	300	310	246
U.S. teachers to foreign countries.............	89	130	97
Foreign teachers to United States.............	11	46	85
Seminars for teachers........................	46	139	293
Technical assistance in education..............	598	768	[1] 800
Study and research..........................	....	....	20

[1] Estimated.

Source: U.S. Department of Health, Education, and Welfare, Office of Education, Bureau of Elementary and Secondary Education.

217-514 O - 66 - 4

Table 17 Students from abroad enrolled in institutions of higher education in the United States: 1959-60 and 1964-65

Major field of interest and year	World total	Far East	Near East	Europe	Latin America[1]	North America[2]	Africa	Oceania	U.S.S.R.	Stateless
TOTAL:										
1959-60	**48, 486**	**17, 175**	**7, 110**	**6, 362**	**9, 428**	**5, 761**	**1, 959**	**568**	**30**	**93**
1964-65	**82, 045**	**29, 400**	**11, 217**	**10, 073**	**13, 657**	**9, 338**	**6, 855**	**1, 265**	**35**	**205**
Agriculture:										
1959-60	1, 615	410	326	161	492	96	98	29	1	2
1964-65	3, 211	935	433	268	649	293	568	64	....	1
Business administration:										
1959-60	4, 114	1, 549	403	495	853	621	150	41	....	2
1964-65	7, 116	2, 528	778	806	1, 350	966	556	121	1	10
Education:										
1959-60	2, 483	866	302	223	403	484	156	46	1	2
1964-65	3, 999	1, 230	451	316	464	1, 029	384	121	....	4
Engineering:										
1959-60	11, 279	3, 642	2, 691	1, 132	2, 520	938	269	53	12	22
1964-65	18, 084	7, 167	4, 117	1, 705	2, 824	1, 069	1, 045	104	4	49
Humanities:										
1959-60	9, 246	2, 706	863	1, 644	2, 050	1, 517	302	139	5	20
1964-65	16, 083	4, 701	1, 430	3, 047	3, 284	2, 435	820	277	15	74
Medical science:										
1959-60	3, 685	1, 236	471	466	738	508	181	64	2	19
1964-65	4, 918	1, 734	549	428	907	659	501	132	....	8
Physical and natural science:										
1959-60	7, 276	3, 344	924	962	889	737	297	107	2	14
1964-65	14, 401	6, 761	1, 787	1, 521	1, 663	1, 240	1, 176	223	7	23
Social science:										
1959-60	6, 782	2, 838	849	1, 048	827	728	399	82	3	8
1964-65	12, 609	4, 012	1, 543	1, 610	1, 987	1, 494	1, 732	202	2	27
All other fields:										
1959-60	482	118	33	29	247	37	17	1	....	....
1964-65	607	154	45	36	281	54	24	12	....	11
No information:										
1959-60	1, 524	466	248	202	409	95	90	6	4	4
1964-65	1, 017	178	84	336	248	99	49	9	6	8

[1] South America, Mexico, Central America, and Caribbean areas.
[2] Bermuda and Canada only.

Source: Institute of International Education, "Open Doors," 1960 and 1965.

"If one were to look for the most significant development in education over the past decade, it would be reasonable to single out the wave of curriculum reform which has swept the school system, and appears to be maintaining its vigor undiminished. Beginning with mathematics and the physical sciences, it has spread in scope until almost every discipline represented in the primary and secondary school curriculum has been in some degree affected.

"These recent reforms have several characteristics that differentiate them from the steady stream of curriculum reform of earlier years. They have been for the most part national, or at least regional, efforts. They have drawn on university scholarship and skilled teachers not only for leadership but for the immediate demands of day-to-day operation; to some extent they have served to destroy (or at least to lower) the wall that has traditionally separated the scholar from the teacher. Almost without exception they have passed from the determination of policy and program directly into the preparation of materials for use in the schools.

"For the most part, they have been eminently successful . . ." [1]

FRANCIS KEPPEL
U.S. Commissioner of Education (1962–65)

[1] "Goals for School Mathematics," report of the Cambridge Conference on School Mathematics, Educational Services Incorporated (New York: Houghton Mifflin), p. vii.

PART TWO/MATHEMATICS EDUCATION IN THE U.S.A.

HISTORICAL BACKGROUND

The successful firing of the first earth satellite, Sputnik I, is considered by many persons as the beginning of the revolution in mathematics education in the United States. This single event, with the roar of an atomic blast, called attention to earlier warnings by feeble voices.

During World War II, the recruitment of young men for the armed services had revealed inadequacies in their mathematical achievement. A few scholars, recognizing their responsibility for this dismal condition, began to participate actively in the beginnings of what has since become a major reform movement.

Even before Sputnik I, some of these persevering individuals had secured limited financial support for research. Among the first to develop a comprehensive program was the University of Illinois Committee on School Mathematics (UICSM), which began experimentation to improve school mathematics curriculum material in 1951. During the late 1940's, some college teachers wrote and tried out course content innovations in their college classes. Sometimes the initiative for such activities came from individuals, sometimes from professional organizations such as the American Mathematical Society, the Mathematics Association of America, and the National Council of Teachers of Mathematics. The learned societies in mathematics had earlier issued joint reports on the need for curriculum revision, but, in general, these pronouncements fell on deaf ears; the conference reports gathered dust on library shelves.

The orbiting satellite in 1957 reminded the American people just how great an explosion of knowledge was taking place in this generation and how much the schools were responsible for preparing youth to cope with technological advances. For the first time, they realized that the explosion of scientific knowledge was changing the economic life, and, in some cases the cultural life, of people in the entire world. The fact that mathematical competency is basic to the social order and that the traditional teaching of mathematics was not meeting the demands of this technological society became a matter of concern and action.

Action was highlighted by (1) the tremendous advances in mathematics research. (2) increased use of automation, and (3) the widespread introduction of computers. With these advances in technology and the awareness that the mathematics curriculum was obsolete, the scholars, teachers, professional organizations, and Federal Government joined to meet the challenge of developing new school mathematics curriculums at every grade level.

CURRICULUM DEVELOPMENTS

For curriculum changes to take place, new materials must find their way through a complicated educational organization into the schools, or they become documents of only historical interest.

On the one hand, the scholar and the Federal Government feel a responsibility for education; on the other hand, the State government and the local school, at least historically, have formulated the educational objectives and selected the course content. Thus, the selection of a new curriculum in mathematics is not done by the Federal Government, but by a local or State school system.

In general, the patterns of curriculum reform in recent years were similar from project to project. First, a group of mathematicians met to review the needed improvements in the mathematics curriculum. Then, usually in subsequent summers, college teachers, research mathematicians, and school-teachers planned course content and wrote sample textbooks. The size of these writing teams varied from as few as 2 or 3 to as many as 40 or 50.

In some cases, the ideas were tried out in the classroom before the material was written; in some, all of the material was written

first; and in others, curriculum development was a combination of both. Materials were used in the classroom before being published as sample textbooks.

Writing teams also produced programed textbooks, teachers' guides, enrichment books, and pamphlets for students; self-instructional units and tests; correspondence courses; supplementary booklets on applications of mathematics to science; and films. Often films, accompanying the printed material, illustrated methods of discovery and pupil involvement with the new content. Other films, showing creative ways of presenting ideas in arithmetic, are available for teacher education.

Some Common Elements in New Mathematics Curriculums

All of the new programs attempt to avoid the presentation of new material as a series of unrelated topics. Instead, they stress unifying themes or ideas in mathematics, such as, structure, operations and their inverses, measurement, graphical representation, systems of numeration, properties of numbers, the development of the real number system, statistical inference, language and notation of sets, logical deductions, and valid generalizations. A comprehensive discussion of unifying ideas appears in the 24th yearbook of the National Council of Teachers of Mathematics, "The Growth of Mathematical Ideas, Grades K–12."

The emphasis on these unifying ideas has resulted in the introduction of words and ideas from college mathematics. For example, the introduction of the notation of sets has involved using symbols and words normally reserved for the college level. It has been the experience of many teachers that the set ideas and language are helpful in explaining many other fundamental mathematical concepts. The structure of mathematics is a basic concept that seems to lend itself to description through the language of sets. Although discovery and an emphasis on the *meaning* of the mathematical operations are not basic mathematical concepts per se, they are characteristics common to the new mathematics programs.

All of the new programs emphasize the structure of mathematics. It is reflected in the careful development of mathematics as a deductive system. The emphasis is on the basic principles or properties common to all systems of mathematics. There has been

a tendency to look at the characteristics of each mathematical model separately. This has resulted in the students learning many seemingly unrelated facts. In the new curriculums, students are encouraged to discover general laws and principles. As an example, they may be asked to consider the following figures:

In figure 1, there are two symbols in the body of the table and a sign of operation in the corner. A means that an object is turned half way around (180°), and B means the object is turned clear around (360°). The → shows the order; A→B=A means the object is first turned 180° and then turned 360°, leaving the object in the same position as if turned 180°:

→	A	B
A	B	A
B	A	B

Figure 1

When the order of turns is reversed, one discovers that:

$$A \rightarrow B = A$$
$$B \rightarrow A = A$$
$$\text{or } A \rightarrow B = B \rightarrow A$$

One sees, then, that the order does not affect the result—a half turn followed by a full turn leaves the object in the same position as a full turn followed by a half turn, and one knows that the commutative property applies for this operation.

It can also be observed that:

$$A \rightarrow B = A$$
$$B \rightarrow B = B$$

or that B has the identity property. In this case, A and B do not represent numbers, and the operation represented by → is not one in arithmetic.

In figure 2, there are again two symbols in the table and a sign of operation in the corner. In this case, the symbols, △ and ○, represent two different abstract ideas; they do not represent horses or dollars.

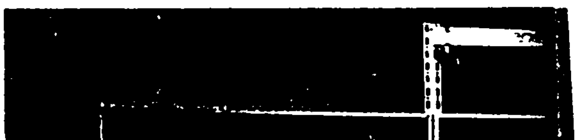
→	△	○
△	○	△
○	△	○

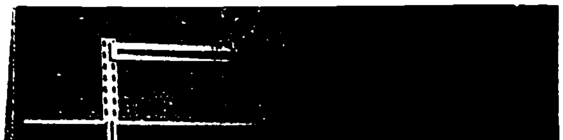
Figure 2

217-514 O - 66 - 5

Concrete to Abstract

Small Group at Work





The $\rightarrow$ represents a rule that is expressed in the figure. That is, if $\triangle$ is paired with $\triangle$, the result is $\bigcirc$, which is written as $\triangle \rightarrow \triangle = \bigcirc$. A study of the table reveals:

$$\triangle \rightarrow \bigcirc = \triangle$$
$$\bigcirc \rightarrow \triangle = \triangle$$
$$\text{or } \triangle \rightarrow \bigcirc = \bigcirc \rightarrow \triangle$$

The order of operation does not affect the result, and this operation has the commutative property.

Also, one sees that:

$$\bigcirc \rightarrow \bigcirc = \bigcirc$$
$$\triangle \rightarrow \bigcirc = \triangle$$

If the circle is used with the circle, the result is the circle; if it is used with the triangle, the result is the triangle. The circle, then, is the identity element with respect to this operation.

Considering both figures, figure 1 is concerned with moving an object, and figure 2 is concerned with abstract ideas. Both have two properties in common. Figure 2 represents a miniature mathematical system; figure 1 is a model or application of the system.

For many years, high school mathematics has consisted of the study of the models, and pupils have failed to see the basic properties common to all. In the improved programs, the pupils look at the mathematical system itself. The properties of the abstract system, figure 2, apply to the models. Properties may be obvious in the abstract system that are hidden by physical objects in the models. The properties of a mathematical system are fundamental and enduring; the models or applications change as the needs of the society change.

ELEMENTARY SCHOOL MATHEMATICS

Discovery and Intuitive Thinking

Some of the new mathematics content for the elementary grades has resulted from attempts to find out if younger children can successfully grasp simple selected geometric and algebraic ideas, which in regular programs are first met at the secondary level. Proponents of these innovations believe that through a discovery approach to learning children often grasp an idea intuitively long before they are ready for the detailed step-by-step analysis of the process. The intuitive approach means a method which yields possible hunches or rapidly formulated ideas, which will later be subjected to more formal analysis and proof. The method implies a freedom to make mistakes and to question. It makes

Learning Mathematics Through Touch

Abstracting From Physical Objects

use of what is known to arrive at a workable procedure as a starting place for solving a problem situation. Important aspects are the "critical question" and a low technical vocabulary. If the child can answer certain key questions, depth of understanding is assumed even though he cannot express his understanding in words.

Geometric and Algebraic Ideas

It is believed that the study of geometry can be expanded far beyond a meager knowledge of shapes, forms, and the computations required for finding areas and perimeters. It may include discovery of the principles underlying area and perimeter and the development of simple concepts with regard to points, lines, and planes in space. From their earliest school experiences with blocks and puzzles, children work informally with shapes and forms. While the lines, points, and planes are ideas, their representations in pictures or in the real environment of the child may be made concrete. In fact, it is possible that simple concepts of geometry are easier for the child to grasp than much of the abstract work with the operations of addition and subtraction, which children have usually been expected to master during their first 2 or 3 years in school.

The introduction of geometry into the early grades (K–3; ages 5–8) has been characterized by two rather distinct approaches. In some programs, the early study of geometry is quite informal beginning with the geometry in the child's environment. As objects are explored and analyzed, the more formal geometric concepts of line, point, and plane are gradually abstracted from the real world of things. Other programs begin with the geometric concepts represented by diagrams and described in precise language. In these programs, construction with straightedge and compass begins around ages 7 or 8. Children are guided as soon as possible to think abstractly with geometric ideas.

Evidence is accumulating from experimental centers to indicate that the mathematics program for children of grades 1–6 may be greatly enriched and broadened by including some simple algebraic ideas. The mathematical sentence in the form of an equation provides a way of getting at the nature of the operations of addition, subtraction, multiplication, and division.

The equation lends itself readily to the use of letters or of frames, such as boxes, triangles, and ovals as placeholders. These are replaced by numerals to make the sentence true. For example, by using frames for the mathematical sentence $\square + 1 = \bigcirc$, children observe that the numeral which replaces the circle is always one more than the numeral which replaces the box and that there are an infinite number of replacements for both. Letters M and N ($M+1=N$) or X and Y ($X+1=Y$) may later be substituted for the frames to emphasize that any number may be used for M and that in this particular equation N will always be one more than M. Students learn that replacements are made in such a way that the symbols on both sides of the equals sign represent different names for the same number. In other words, if the box is replaced by 3, the oval should be replaced by 4 to make the sentence true. The number sentence is $3+1=4$; $3+1$ and 4 name the same number.

Children learn that sometimes a true sentence can result if one and only one numeral is selected. The only replacement for the box in the equation $\square+3=7$, which will result in a true statement, is 4. Sometimes several replacements are possible. The frames in the equation $7=\square + \bigcirc$ may be replaced by $3+4$, $4+3$, $1+6$, $6+1$, $2+5$, $5+2$, $0+7$, and $7+0$. When the child learns about fractions, many other replacements are possible. Sometimes none of the numbers children know will work. Replacement for the frame in the equation $\square+4=2$ must wait for an understanding of negative numbers.

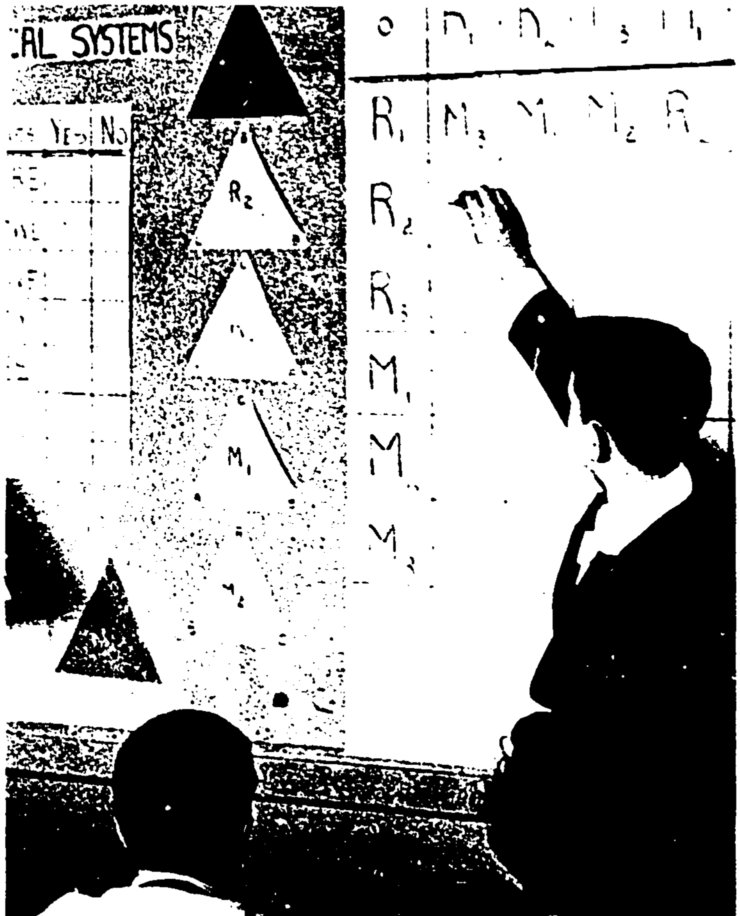
Operations and Properties of Numbers

Subtraction and division are presented as the "undoing" of addition and multiplication, respectively. Subtraction reverses the action associated with addition or brings the elements back to their original condition. In like manner, division reverses the action associated with multiplication.

Children learn that zero is a number having special properties. It is a number that designates not any. When it is added to a number, the same number results. When it is subtracted from a number, the same number results: $3+0=3$ or $X+0=X$, $4-0=4$ or $X-0=X$. It is known mathematically as the identity element for addition, because, when zero is added to a number, the sum

Discovering Properties

Finding Ratios



is the number itself. Children learn that one is the number in multiplication which serves a similar purpose to zero in addition. It does not change the product: $8\times 1=8$ or $X\times 1=X$.

Principles involved in the commutative, associative, and distributive laws of mathematics are a fundamental part of the newer experimental programs. Through the application and understanding of these principles, children are assisted in developing not only skill but also concepts of the nature of the operations, appreciation for the flexibility which is possible in mathematics, and understandings underlying the algorithms or forms of recording mathematics. The significance of these laws and properties of mathematics becomes clear to children who have opportunities to discover them and to observe what happens as they are used. Their application may be made simply and concretely in the early stages of learning and extended as advanced mathematics is experienced in later grades. The definition and illustrations of the distributive property serve as reminders of the laws and of some of their applications to the mathematics of the elementary school child.

Definition of Distributive Property: $a(b+c)=(a\times b)+(a\times c)$

Illustration 1: $2(3+4)=(2\times 3)+(2\times 4)$
$2\times 7=6+8$

Illustration 2:
$$\begin{aligned} 8\times 36 &= 8\times(30+6) \\ &= (8\times 30)+(8\times 6) \\ &= 240+48=288 \end{aligned}$$

Illustration 3:
$$\begin{aligned} 36\times 24 &= (36\times 20)+(36\times 4) \\ &= (30\times 20)+(6\times 20)+(30\times 4)+(6\times 4) \\ &= 600+120+120+24=864 \end{aligned}$$

Extensions of Familiar Topics

Other extensions of familiar topics expand the study of our decimal numeration system to include an introduction to negative numbers and powers in the upper elementary grades. An introduction into bases other than 10 may provide interest and variety, but it is intended primarily to promote a deeper understanding of our decimal numeration system. The emphasis on relationships within and between processes is continued as children are encouraged to

reason from known to unknown facts: if $5+5=10$, then $5+6=11$; if $10+6=16$, then $9+6=15$; if $10\times6=60$, then $9\times6=60-6$ or 54; if $3+9=12$, then $12-3=9$ and $12-9=3$; if $4\times6=24$, then $24\div6=4$; if $\frac{1}{4}\times\frac{2}{3}=\frac{1}{6}$, then $\frac{2}{3}\times\frac{1}{4}=\frac{1}{6}$ and $\frac{1}{6}\div\frac{1}{4}=\frac{2}{3}$ and $\frac{1}{6}\div\frac{2}{3}=\frac{1}{4}$, etc.

Teachers of the early grades are familiar with a study of "groups" as background for the arithmetic operations of addition, subtraction, multiplication, and division. Some of the newer programs are using the concepts of sets and the terminology of sets as a more mathematically accurate introduction to these operations. A "set" is a well-defined collection of objects, which are not necessarily alike in any way; for example, a triangle, a square, and a circle; and a balloon, a cart, and a jump rope. In each example, there are three things. The objects of one set can be matched one-to-one with the objects of the other. The triangle can be matched with the balloon, the square with the cart, and the circle with the jump rope to show that these are equivalent sets. Both have the same cardinal number. The numeral "3" names the number of objects for either set. The objects in two sets may be combined to make a third set. A set may be separated into two subsets. One set may be compared with another. A set may be used repeatedly or divided into subsets having the same number of elements. These procedures of identifying, combining, separating, and comparing when performed with numbers are roughly analogous to the operations of addition, subtraction, multiplication, and division with numbers.

Still other extensions and applications of familiar topics include (1) a greater emphasis on the understanding of large and small numbers to help children to better meet today's needs, (2) different ways of arriving at the same answer, (3) more emphasis on estimation and mental arithmetic as essential for the numerical thinking demanded by present society, and (4) variety in practice procedures required for maintenance of skills which will foster interest and provide new learning at the same time.

Summary

The decision to introduce advanced mathematical concepts at an earlier age than formerly has necessitated a search for methods appropriate to the child's level of understanding. For example, the use of open frames, such as boxes, triangles, and diamonds

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instead of letters, has made it possible for children in the first 3 years of school to deal successfully with the concept of variables. Early use of the number line has enabled children to progress rapidly from a study of positive whole numbers to the negatives and rationals. Recent programs are emphasizing a laboratory approach to teaching mathematics and the individualization of instruction. The innovations which characterize experimental programs have had a great influence on the content of mathematics textbooks prepared for distribution by commercial publishers.

Secondary School Mathematics

Present society is demanding more and better trained scientists and technicians; it is requiring that citizens have mathematical knowledge.

To meet these demands of a rapidly advancing society, the leaders in mathematics education have agreed that (1) the mathematics program for the potential scientist and technician must be improved, (2) the mathematics program must attract and hold more non-scientific students, and (3) teachers must be prepared to teach these courses.

Evolving programs in elementary and secondary schools have radically changed the preparation of high school students for college. A decade ago the secondary school pattern quite generally was 9th grade algebra, 10th grade geometry, 11th grade algebra, and one-half year each of trigonometry and solid geometry in the 12th grade. Each class met approximately 1 hour a day, 5 days a week, and a particular subject was studied throughout an entire semester or year.

As a result of the report of the Commission on Mathematics of the College Entrance Examination Board in 1959 and the experimental curriculum projects such as the School Mathematics Study Group, the program in secondary school mathematics is now generally as follows:

School Year 7. Study of an informal structure of whole numbers; the algorisms for computation; the positive rational numbers; applications to percent, distance, area, and volume; informal geometry including constructions; and introduction to algebra.

School Year 8. Study of an informal structure of the rational numbers; the Pythagorean theorem; irrational numbers; the real

number line; negative numbers; solution of simple equations and inequalities; finite number systems; graphs and statistics; indirect measurement and numerical trigonometry; introduction to deduction and proof.

School Year 9. A year of study of elementary algebra from a more formal study of number systems; sets and set operations; operations on polynomials and rational expressions; solution of equations and inequalities; use of deduction and proof.

School Year 10. A year of strong deductive axiomatic geometry, plane and solid, using the properties of real numbers; the introduction and use of rectangular coordinates.

School Year 11. A year of extended study in algebra, including a formal study of the system of real numbers: linear, quadratic, rational, exponential, logarithmic, angle, and circular functions; inverse functions; the related algebraic solution of equations and inequalities and transformations of these functions.

School Year 12. Although the work of this year is not standardized, the course usually includes a continuation of the study of algebraic functions, limits and continuity, probability and statistical inference; matrix algebra, including simple vector spaces; and an extension of trigonometry and solid geometry.

For the abler students, the above program is completed by the end of School Year 11. The 12th year is then usually devoted to a study of analysis, including analytic geometry and differential and integral calculus. These students then take the Advanced Placement Examination of the College Entrance Examination Board whereby they may receive college credit and/or advanced placement depending on the policy of the particular college. Changing college programs, particularly in the freshman year, have focused the attention of secondary schools on preparing their students to make the transition to college study.

Mathematics Curriculum for the Nonacademic Student

Although much attention is given to students who do college preparatory or advanced-placement college work in high school, schools are making a great effort and are doing much experimentation in an attempt to develop the most appropriate mathematics curriculum for students who are not prepared for the college preparatory sequence of courses or who do not choose to enroll

in them. Various types of "general" mathematics courses are usually offered for such students. They are designed to be partly remedial, to give students some new mathematical experiences, to prepare students to solve the problems of their jobs and their daily lives, and to strengthen them to enter the college preparatory sequence if they choose to do so.

Many schools require for graduation only 1 year of mathematics beyond grade 8 (age 13 or 14). However, a second year (and occasionally a third and fourth year) of general matehmatics is usually offered.

Numerous groups have prepared curriculum materials for such courses. For instance, the School Mathematics Study Group has rewritten the junior high school mathematics courses to make them more suitable for slower students. The elementary algebra course has been rewritten as a 2-year course, so that students who cannot achieve success in a 1-year course can do well at this slower pace.

The National Council of Teachers of Mathematics considered the problem of teaching mathematics to slower students so critical that it sponsored the writing of units for slow ninth grade students which are being tried out and evaluated in many schools. Since many high school dropouts come from the group that is not taking the college preparatory courses, schools are making special efforts to locate or develop new curriculums, which help to motivate them to stay in school until they learn some salable skill.

Selection of Textbooks

The preparation of teaching materials is only a first step. After textbooks are written, the task of introducing them into the schools is not a simple one. Textbooks are not selected by a Federal agency or even by each State agency. The local school units select the textbooks for both the elementary school and for the high school in about half of the States. In cases where the State makes the selection, the local school unit is usually permitted a choice among several textbooks. In only a few States is a single textbook adopted for any course. In more than half of the States, schools are required to use the same textbooks for a specified number of years. The time varies from 3 to 8 years. Thus, for a textbook to find its way into a school, (1) a new textbook must be under consideration (in many States this happens only every 5 or 6 years);

(2) the local textbook selection committees (there are hundreds of such committees in the United States) must become familiar with new programs and be convinced of the desirability of adopting a new one; and (3) textbook committee recommendations usually must be approved by the school authorities before being available for purchase by the schools.

Curriculum Enrichment

Besides the regular mathematics courses in secondary schools, many activities are provided which enrich the academic growth of students, motivate them to further study, stimulate them to individual investigations, and develop leadership. The activities vary greatly but some typical provisions for enrichment are:

Mathematics clubs, including Mu Alpha Theta (MATh) a national high school and junior college mathematics club.
Mathematics institutes sponsored by colleges.
Mathematics contests and tournaments.
Paper reading competitions.
Fairs and congresses where students exhibit original work or present lecture-demonstrations.
Courses in electronic digital computing.
Lectures and seminars.
Science centers.
Summer honors programs.
Summer mathematics camps.
Science fairs exhibiting pupil projects which were constructed to illustrate mathematical or scientific principles.

College Mathematics

All levels of the mathematics curriculum are undergoing extensive revisions, and the undergraduate level is no exception. These revisions of the undergraduate program have been encouraged by the following facts:

1) Students are entering college with a better preparation in mathematics.
2) The increased complexity of science and technology calls for more mathematics. Engineers, as well as scholars in other disciplines, are finding that they need unfamiliar mathematics. This leads to the introduction of new aspects of mathematics as well as to the offering of a greater variety of courses.

3) There is an increased use of electronic computers. The result has been more college courses in mathematics.
4) The number of students graduating with majors in mathematics is steadily increasing. With the expansion in mathematics enrollments, a diversification has come in kinds of mathematics offered.

Many mathematics departments have revised and developed individual courses. However, the greatest single attack on the undergraduate program has been by the Committee on the Undergraduate Program in Mathematics, a committee appointed by the Mathematical Association of America and supported in part by the Federal Government. The committee has recommended the following minimum undergraduate program in mathematics, but many colleges are now exceeding these basic recommendations:

13th and 14th year (18- and 19-year-old students)

1. Calculus	3. Mathematical Analysis
2. Probability	4. Linear Algebra

15th and 16th year (20- and 21-year-old students)

1. Multivariable Calculus	5. Geometry
2. Algebraic Structure	6. Applied Mathematics
3. Probability and Statistics	7. Theory of Real Variables
4. Numerical Analysis	8. Complex Analysis

In the 15th and 16th year, courses 4 and 7 meet for 1 hour 3 days each week for the school year. All other courses are half-year courses. Course 2 is not required of all students. The student is asked to take at least 6 half-year courses in the 15th and 16th years.

A detailed outline of each course has been prepared. It is expected that sample textbooks based on these outlines will be published in the near future.

To provide for the large influx of undergraduate students, mass production teaching, such as TV, teaching machines, and large lecture sections, are being used. None of these seems to be entirely satisfactory.

EQUIPMENT AND LEARNING AIDS

The use of instructional aids in the teaching of mathematics at both elementary and secondary education levels in the United

States has been accelerated during the past several years for various reasons:

1) Numerous nationwide curriculum improvement efforts have been supported by both Federal and philanthropical funding.
2) The introduction of contemporary mathematics courses into the curriculum has placed added emphasis on the "why" of mathematics, which can often be illustrated best by learning aids.
3) Federal programs have assisted schools in obtaining learning aids since mathematics is considered one of the critical subject areas demanding special attention.
4) Commercial concerns have developed equipment designed to create a laboratory setting in the mathematics classroom.

The U.S. Office of Education administers a program through which Federal grants are made to State educational agencies to aid local public schools in acquiring specialized equipment and materials for strengthening instruction in mathematics and eight other critical subjects. During the first 7 years of this program, State or local educational agencies have matched over $30 million [1] in Federal funds for the purchase of instructional equipment in the field of mathematics. Purchases for both elementary and secondary levels of education have run the gamut from numberline aids to electronic computers and have included such items as reference books, counting devices, geometric models, calculators, place value devices, aids for explaining fundamental operations, area and volume measurement devices, and overhead projectors. These instructional aids have enabled many teachers to utilize, as never before, a laboratory or discovery approach to the teaching of mathematics—leading students to discover mathematical relationships for themselves.

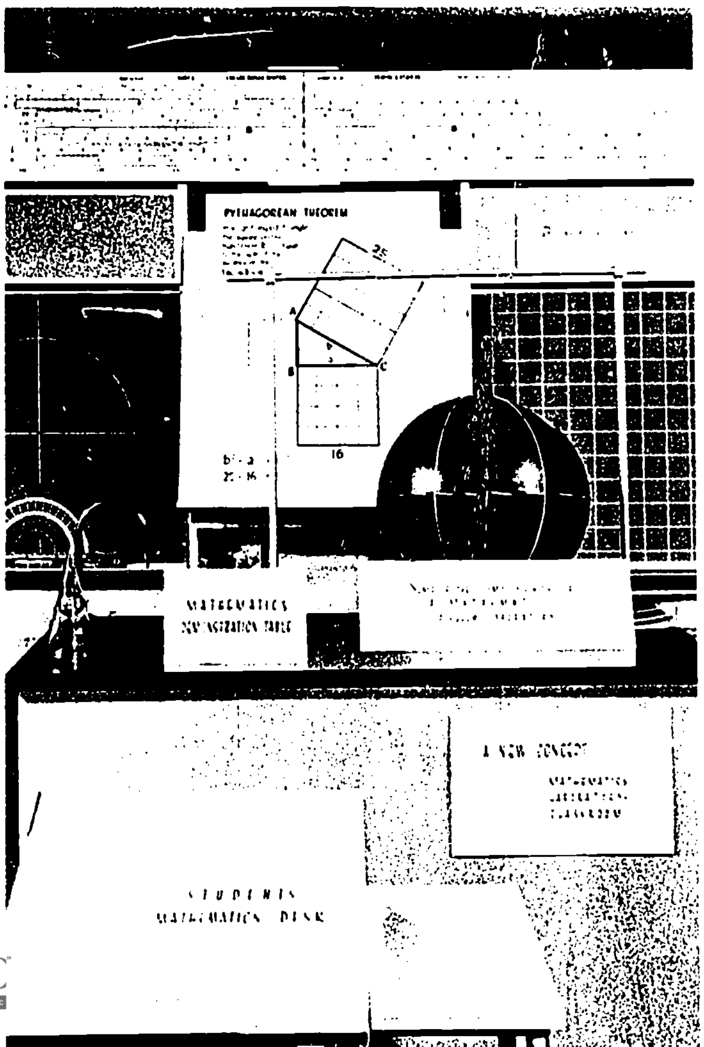
The Mathematics Laboratory

Beads, blocks, charts, number lines, and similar concrete and pictured materials have been used by elementary school teachers for many years. Recently, a few school systems have provided a supplementary classroom or laboratory amply equipped with a

[1] Based on statistical report for seven fiscal years, 1959–65: 96,657 approved projects for specialized mathematics equipment and minor remodeling cost an estimated $67,382,134 in Federal, State, and local funds.

Mathematics Laboratory Equipment

Integrating Science and Mathematics



variety of learning aids under the direction of a teacher. At the secondary school level, the laboratory is much more widely used. It is estimated that there are currently around 2,000 well-equipped mathematics laboratories in the U.S.A. The elementary laboratory contains such materials as objects for counting, combining, and separating, materials for measuring, models for study, calibrated rods, place value charts, hundred-boards, coordinate charts, decimal and percentage charts, globes, maps, and scales. While the secondary school laboratory has some of these same materials listed for the elementary laboratory, it also includes more complex geometry models, chalkboard geometry sets, materials for construction, and slide rules. Both elementary and secondary school laboratories are equipped with overhead projectors with prepared transparencies and materials for the preparation of other transparencies as needed, plus films, film strips, and projection equipment. Books, workbooks, worksheets, progress charts, and similar materials are valuable resources in any mathematics laboratory.

The laboratory serves the purpose of encouraging pupils of all levels of ability to explore and discover mathematics under the guidance of a well-qualified mathematics teacher. It stimulates the able child, opens new avenues of experience for him, and enables him to progress as rapidly as he is able. The slow student profits from the concrete aids the laboratory provides and is allowed to work at his own pace in reaching abstract levels of thinking. The laboratory method encourages the teacher to individualize instruction, to provide the means whereby proofs may be obtained, and to stimulate pupils to strive continously for higher levels of attainment. It is necessary for the laboratory teacher to keep careful records on each pupil's progress.

Although there is a lack of space and equipment for a laboratory in many schools, teachers at both the elementary and secondary level circumvent this difficulty by making use of the laboratory method in their own classrooms for a part of the time devoted to instruction in mathematics.

EVALUATION

The ideal of having every student achieve the maximum of his potential is one to which American education is committed.

Evaluating the attainment of this ideal is difficult and is accomplished in many ways.

Evaluation—Elementary Schools

Evaluation is an essential part of instruction and learning in a good elementary mathematics program. More important than the simple measurement of a correct or an incorrect answer is the evaluation of thought processes of the child. While it is possible to obtain limited information on levels of thinking by means of pencil and paper tests, these are inadequate means of determining at what point in learning the difficulty occurred or what procedures led to the answer obtained. In fact, there is probably a stage in the learning of a concept when a relatively mature response ending in an incorrect answer may result in more correct answers for the future than a correct answer lacking depth of understanding, which can have no transfer value to similar situations.

Through such techniques as observation of the child at work and informal interviews with him, the teacher can gain evidence and information to guide the next steps in the learning process. The teacher who frequently challenges the child as a regular part of any mathematics lesson, by such questions as Why? How do you know? Could you prove that your answer is right? Is there another way to find the answer? Uses evaluation as a part of the teaching process.

Recent strides in computer assisted instruction now make it possible to provide practice, to obtain an immediate summary of errors and correct answers, time record, and a list of items missed for self-evaluation by the child and follow-up by teacher or parent. To the extent that automation reduces the amount of time the teacher must spend on correcting routine drill exercises, more time will be available for complex evaluation procedures, such as observation, interviews, and the interpretation of data, which require the teacher's insight and knowledge.

In addition to the evaluation by the teacher of his own program, there is a need for evaluation of the mathematics program of the entire school system by representative teachers, principals, supervisors, and others competent to assist in the process. All available information obtained from test measures and from an analysis of children's successes and difficulties, together with value judgments

arrived at from a study of the literature in the field, will be used to arrive at workable evaluations.

At the present time, school personnel must rely on available objective data, supplemented by value judgments, to answer such important questions as:

1) What is the relative importance of speed and accuracy at various stages in the learning of a concept?
2) What mathematics can be taught with the greatest conservation of teaching and learning time at a given period?
3) Among all the mathematics that 5–8-year-olds *can* learn, which is most appropriate or most necessary for successful living and for future progress in mathematics?
4) On the basis of available evidence, what changes are indicated in the selection and arrangement of topics within sequences?
5) What changes would provide more adequately for meeting the individual needs of pupils?
6) What evidence is there that the child can reason, use the process of discovery, apply generalizations to new problems, and approach problem situations creatively?

Standardized tests will continue to be useful in measuring some aspects of any mathematics program. In particular, these tests measure certain types of computational skills which are included in the objectives of all mathematics programs. Such important objectives as the child's ability to apply mathematics to other situations and his attitudes toward and interest in mathematical experiences can be evaluated in part by the use of rating scales and by the subjective evaluation of school personnel. Improved techniques of measurement will make it possible to evaluate more completely the results of the newer projects.

Evaluation—Secondary Schools

After individual school systems decide on their objectives and curriculums, the measurement of progress is made in terms of these. Talents and weaknesses are diagnosed, sometimes with the help of standardized tests. Students are placed in appropriate sections or courses.

Evaluation of progress is then done in some of the following ways. Teachers observe evidence of growth in students' oral and written work. They help and encourage students to evaluate themselves

Observing Children Learning

Self-Testing

Teachers prepare and administer tests, which give evidence of accomplishment of students toward the goals which the school has set up. Such evaluation is often supplemented by standardized tests which are prepared by commercial publishing firms. These tests are made available for purchase by schools only after test questions have been selected on the basis of thorough analysis. Scores on these tests are then used for such purposes as supplementing teacher judgment in assigning grades or in passing or failing students, serving as a basis for guiding students into the next mathematics course, and giving the school officers information about progress that students are making. Although school curriculums are usually local agreements, standardized tests are based on what is taught nationally. Thus, there is rather general uniformity in the content of specific courses, especially in college preparatory mathematics.

With the advent of new mathematics curriculums, the problem of evaluation was particularly difficult. New programs necessitated tests which evaluated achievement in the goals of those particular programs. School administrators and parents, as well as the teachers, wanted to be sure that students could do satisfactorily on traditional tests. They have been satisfied with new programs, because students showed that they can perform well on such tests as well as learn the new subject matter.

Longitudinal studies are now underway to learn the long-term effects of some of the new mathematics programs. Answers are being sought to such questions as: Have the students continued their study of mathematics beyond the required courses? Have they elected mathematics courses in college? Have they pursued scientific careers? How does their college achievement in mathematics compare with students who studied traditional mathematics courses?

One widely used standardized test is the so-called College Entrance Examination. Many colleges require that applicants take this examination so that the score can be considered when the application is processed. One examination is a 3-hour Scholastic Aptitude Test (SAT). Approximately 1,350,000 candidates took the SAT during the 1964–65 school year. The SAT is designed to provide reliable indications of a student's ability to do college work. It is an objective (multiple-choice) test. It provides college admis-

sion officers and high school counselors throughout the country with a common measure of ability. Additional information is provided by the subject matter achievement tests. These are widely used, as shown by the fact that they were taken by 530,000 candidates in 1964–65. The mathematics examination is either Mathematics Level I (Standard) or Mathematics Level II (Intensive). Each is a 1-hour objective type test.

With the appointment of the Commission on Mathematics by the College Entrance Examination Board in 1955, there was an attempt to influence the mathematics curriculum to meet the challenge of the dramatic developments in mathematics and technology. There followed a gradual but steady shift in the content of the examinations toward more contemporary emphases. This shift is still in process.

Following are several illustrations [1] of some of the newer types of questions, which are designed to test understanding and the ability to apply concepts rather than merely to recall facts or apply rules.

1. If there are 400 students in a school, which of the following statements is (are) true?
   I. There must be at least 1 month in which 30 or more students have a birthday anniversary.
   II. Some students must have birthday anniversaries on the same day.
   III. Some students must have been born in the same year and on the same day.
   (A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III
2. Two variables in a scientific experiment are such that their product is always 1. If, for a certain time, one variable is greater than zero, less than 1, and decreasing, then which of the following describes the second variable?
   (A) Greater than 1 and increasing
   (B) Greater than 1 and decreasing
   (C) Not changing
   (D) Less than 1 and increasing
   (E) Less than 1 and decreasing

[1] "Questions Illustrating the Kinds of Thinking Required in Current Mathematics Tests." Selected by Dr. Sheldon S. Myers, Educational Testing Service, Princeton, N.J., 1964.

3. If n is an integer and if the following are arranged in order, which integer is in the middle?

(A) $n+3$ (B) $n-9$ (C) $n-4$ (D) $n+6$ (E) $n-1$

4. If $\emptyset$ is an operation on the positive numbers, for which of the following definitions of $\emptyset$ is $x\emptyset y=y\emptyset x$?

(A) $x\emptyset y=\frac{x}{y}$

(B) $x\emptyset y=x-y$

(C) $x\emptyset y=x(x+y)$

(D) $x\emptyset y=\frac{xy}{x+y}$

(E) $x\emptyset y=x^2+xy^2+y^4$

5. Which one of the following must be *excluded* so that the remaining four are consistent?

(A) $a>b$ (B) $a>d$ (C) $b>c$ (D) $c>a$ (E) $d>c$

6. If the average of 13 consecutive whole numbers is odd, then the product of the first and last of these numbers must necessarily be

(A) odd (B) even (C) a multiple of 7 (D) a multiple of 13 (E) a multiple of the average of the 13 numbers

7. How may numbers in the set $\{-5, -3, 0, 3\}$ satisfy both of the conditions $|n-3|\leq 6$, and $|n+2|<5$?

(A) None (B) One (C) Two (D) Three (E) Four

Evaluation—College

There are definite indications that improved mathematics education in the secondary school is having an impact upon colleges and universities. Comparisons made from a comprehensive survey in 1960 and a smaller follow-up study in 1965 show that students are entering 4-year colleges today with better preparation in mathematics than 5 years ago.

Whereas less than a quarter of entering freshmen were enrolled in analytic geometry, calculus, and courses of equivalent level and above in the fall of 1960, more than a third of entering freshmen were enrolled in these courses in 1965. Enrollment in the lowest

Answers: 1. D, 2. A, 3. E, 4. D, 5. D, 6. A, 7. C.

level courses (courses preceding college algebra and trigonometry) showed a drop of almost half the number of students in 1965 as compared with 1960.

These facts are corroborated by judgments made by the respondents to the 1965 survey, who were asked if they had been able to detect any improvement in the mathematics preparation of entering freshmen as a result of the new elementary and secondary curriculum materials in mathematics.

Impact of New Curriculum Materials

	Type of institution			
	Universities	Liberal arts colleges	Teachers colleges	Technological institutions
Number of institutions responding to question	45	90	20	15
Have the new curriculum materials in elementary and secondary schools brought out better preparation in mathematics among entering freshmen?				
Yes	28	70	12	5
No	12	18	7	8
No opinion	5	2	1	2

The table shows that except for the small number of technological institutions, the respondents were overwhelmingly of the opinion that the new curriculum materials in elementary and secondary schools were causing the improved preparation in mathematics among entering freshmen in baccalaureate granting institutions. A majority of the respondents felt that entering students were able to take more advanced courses than formerly and that they had better understanding of the concepts and structure of mathematics. Several of the respondents declared, however, that although the students generally had better understanding of mathematics, they were not as good in manipulative skills as formerly.

The 1965 survey brought out some other facts about the impact of the new mathematics programs in elementary and secondary schools upon the colleges. Respondents were asked if during the

past 5 years they had significantly altered the freshman-year program. They were also asked if they had introduced new or substantially altered programs of teacher preparation for secondary school teachers and for elementary school teachers. These responses are significant because they reflect the advancing levels of entering freshmen and the need for teachers who are qualified to teach the new curriculum materials in the schools.

Innovations in College Programs Related to New Curriculum Materials in the Schools

	Type of institution			
	Universities	Liberal arts colleges	Teachers colleges	Technological institutions
Number of institutions responding to question	48	94	20	15
Have significantly altered the freshman-year program	28	51	14	8
Have introduced a new program or substantially altered a previous program for the undergraduate preparation of secondary school teachers	24	46	10	2
Have introduced a new program or substantially altered a previous program for the undergraduate preparation of elementary school teachers	18	41	14	2

Hence, data strongly indicate that the new curriculum materials in mathematics in the elementary and secondary schools have resulted in improved preparation of high school students who are entering baccalaureate institutions. Neither the 1965 or 1960 surveys, however, involved junior colleges which are becoming a growing segment of the total freshman and sophomore undergraduate population in the United States. The possibility exists that many of the 4-year colleges are raising their standards under the pressure of expanding enrollments, resulting in the less-well-prepared students going to the junior colleges. Nevertheless, improvement in the preparation of entering freshmen in 4-year colleges is sufficiently pronounced to justify the conclusion that

students as a whole are graduating from American high schools today better prepared in mathematics than they were in 1960.

TEACHER EDUCATION

"The education of teachers must guarantee high scientific and pedagogical knowledge that leads to continued professional study throughout their entire career" concluded the conference at an International Working Session on New Teaching Methods for School Mathematics.[1] In order to prepare U.S. teachers to teach the new programs, it became necessary to organize both short and long term programs.

The mathematics curriculums which have been developed in the last few years have emphasized greater understanding of mathematical processes; they have given more attention to the structure of mathematics; and they have introduced many new topics and courses which until recently were not taught in the schools. Teachers who fulfilled all the requirements for certification a few years ago found themselves unprepared for the new emphases in mathematics. Greater attention had to be given to the undergraduate preparation of mathematics teachers, to inservice education, and to the preparation of suitable teachers' materials to accompany new curriculums.

Preservice Education

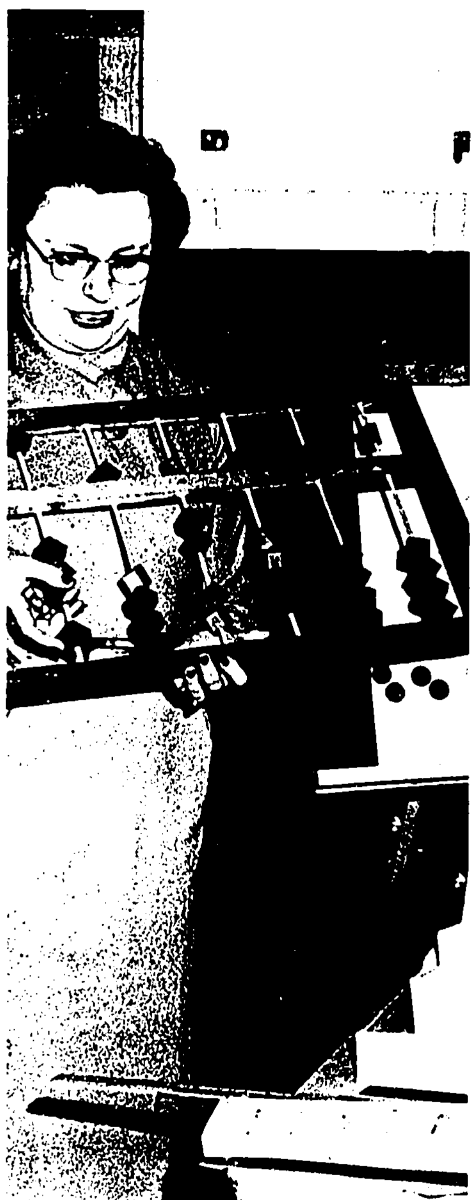
The professional organization which is well suited to attack the problem of preservice training is the Mathematical Association of America, the membership of which is largely college teachers of mathematics. This organization set up the Committee on the Undergraduate Program in Mathematics (CUPM), supported in part by the National Science Foundation, to develop a broad program of improvement in the undergraduate mathematics curriculum of the Nation's colleges or universities. As part of its mandate, CUPM established a Panel on Teacher Training, which was to prepare a set of recommendations of minimum standards for the training of teachers on all levels. This Panel made recommendations for teachers on five levels:

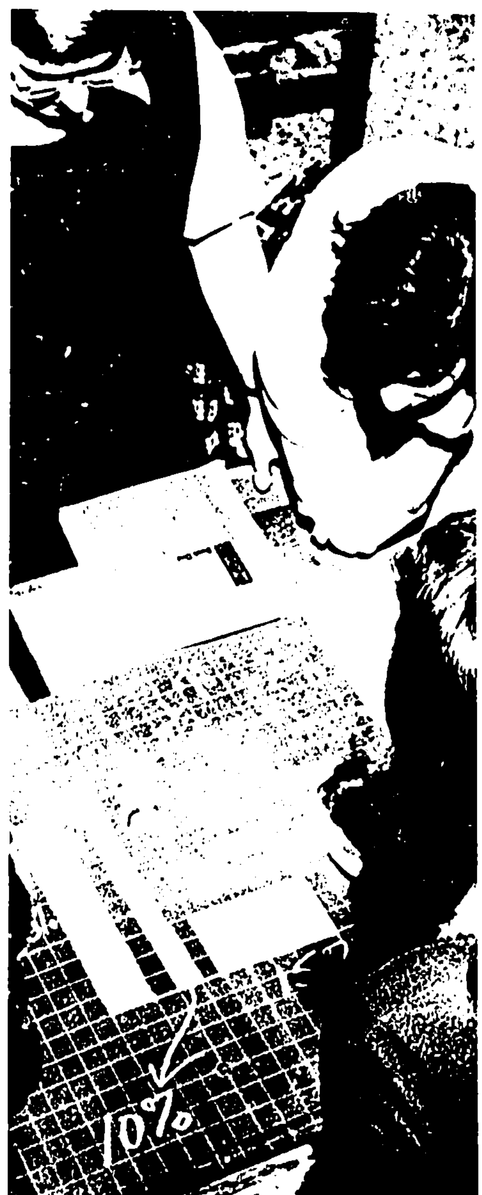
[1] The conference was held in Athens, Greece, Nov. 17–23, 1963, under the auspices of the Organization for Economic Cooperation and Development.

Using a Calculator

Aids to Teaching

Simple Learning Aids





Level I. As minimum preparation for teachers of elementary school mathematics (grades K–6; ages 5–11), the Panel recommended that the students have as a prerequisite at least 2 years of college preparatory mathematics, consisting of a year of algebra and a year of geometry, and that the college training consist of the equivalent of the following courses: a two-course sequence devoted to the structure of the real number system and its subsystems, a course devoted to the basic concepts of algebra, and a course in informal geometry.

Level II. Prospective teachers of the elements of algebra and geometry (grades 7–10; ages 12–15) should enter the college program ready for a course at an equivalent level of a beginning course in analytic geometry and calculus, which would require a minimum of 3 years of college preparatory mathematics. The college training should then include three courses in elementary analysis and four other courses: abstract algebra, geometry, probability from a set-theoretic point of view, and one elective. One of these courses should contain an introduction to the language of logic and sets.

Level III. Prospective teachers of high school mathematics (grades 9–12; ages 14–18) should complete a major in mathematics and a minor in some field in which a substantial amount of mathematics is used. The minimum preparation should consist of three courses in analysis, two courses in abstract algebra, two courses in geometry beyond analytic geometry, two courses in probability and statistics, and two electives; e.g., introduction to real variables, number theory, topology, history of mathematics, or numerical analysis including use of high-speed computing machines.

Level IV. The Panel recommended that teachers of the first 2 years of college mathematics should have a Master's degree with at least two-thirds of the courses being in mathematics and for which an undergraduate program as strong as Level III training is a prerequisite. Since these teachers may be called upon to teach calculus, the program should include the equivalent of at least two courses of theoretical analysis in the spirit of the theory of functions of real and complex variables.

Level V. For full college mathematics teaching additional preparation is necessary. The present shortage of qualified college mathematics teachers is creating tremendous problems.

Inservice Education

Teachers, who enter the profession with minimum preparation and those whose preparation is not recent, find that they need a broader and deeper knowledge of mathematics and more familiarity with all phases of mathematics education. These needs are met in countless ways, ranging from individual reading to sabbatical leave for a full academic year of study at a college or university.

The last decade of Americans have witnessed the largest back-to-school movement in history. The newer aspects of arithmetic, algebra, geometry, and analysis were taught in thousands of classes held on Saturdays, after school hours, and/or during holidays. Some of the inservice work has been through college courses for credit or courses planned especially for teachers as in summer and inservice institutes, but many courses have offered no college credit.

Mathematics consultants or supervisors at the State level have sponsored widespread inservice offerings in a systematic effort to provide the opportunity for every elementary and secondary school teacher to upgrade his teaching competence. Ordinarily these courses do not carry college credit but enable the teacher to acquire professional credit at the local level. Inservice courses vary from a concentrated 1 or 2 weeks workshop to half- or full-year courses provided in weekly sessions.

Local school systems have also been active in sponsoring inservice work, which is planned especially for the particular system. Instructors for inservice courses are drawn from the locality and from nearby colleges and universities.

The urgency of the need for the continuing education of mathematics teachers led the National Science Foundation (NSF), an agency of the Federal Government, to develop many ways of providing such education. This agency provides summer and inservice institutes for elementary, secondary, and college personnel. For secondary school personnel, it also provides academic year institutes, research participation, and cooperative college-school science programs. For college teachers, NSF also finances research participation and a "visiting scientists program." In 1966, about 36,000 teachers will study in these various programs. More than $100 million of tax money has been spent on NSF mathematics institutes for secondary school teachers alone.

Much professional growth takes place in inservice activities. which do not offer college credit. Some of these are departmental meetings at which current topics and problems are discussed, attendance at demonstration lessons and participation in the followup discussions, and courses or lectures via television. Much inservice education takes place in curriculum experimentation which may involve writing units of work, trying them out in the classroom, evaluating and revising them, and communicating results in faculty meetings and meetings of professional organizations. Other valuable committee work includes the establishment of criteria for selecting textbooks, films, and learning aids. Classes in the techniques and implications of electronic computation help teachers to understand the impact of computers in this modern age and also to teach such courses to students.

The many inservice activities and the improved preservice programs have greatly helped the basic task of preparing teachers for a life of continued self study. However, positive action and research must continue.

RESEARCH IN MATHEMATICS EDUCATION

The new elementary mathematics programs present many topics and terms that traditionally have been reserved for more advanced students. It is natural for teachers to question the ability of young pupils to understand these concepts and terms, and recent research has reflected their concern. The research studies in mathematics collected biennially by the U.S. Office of Education indicate that many young children can learn more mathematics than has been expected of them, but, because of differing abilities and differing backgrounds, not all of them can learn the same things at the same age. Thus, the crucial question seems to be: What mathematics should what children learn at what age? Experimentation and research are underway to arrive at reliable and valid answers.

Aids in effective learning were also a subject of inquiry in several research studies, and, similarly, the crucial unanswered question was: Which aids for which children and for which topics? Most research shows that multisensory aids are helpful to some pupils, but the research must continue to determine direction concerning

Research in Nonvisual Learning

Forms Aid Teaching

which students and with which concepts or at which stage of conceptual development a specific aid should be used.

Grouping of pupils seems to increase learning in certain topics with some pupils, but the question—With what topics and pupils can it be helpful?—remains unanswered.

Research indicates that the concepts discovered by the pupils are retained longer than concepts presented formally. Teaching with an emphasis on meaning of concepts and operations permits many pupils to apply their knowledge more readily. Although many studies on the best way to teach a skill or concept have been conducted, the research has not yielded great returns. Perhaps, there is no one best method. The recent research studies show that the new mathematics programs result in more effective learning of mathematics by, at least, the superior elementary school pupil.

Many of the recent research studies on high school mathematics are directly related to the new mathematics programs.

The majority of these studies have compared, by means of traditional tests, the achievement of pupils in the new curricula with those in traditional courses.

Tests showed that the pupils in the new programs did learn traditional material. There is also evidence that pupils in the new programs learned material that the other pupils did not have an opportunity to learn.

Research shows that high school students can learn many concepts usually reserved for college students. Many mathematicians enthusiastically recommend that the "new" topics be introduced into the high school program. Others strongly discount the value of these "new" topics for high school youth. The decision to introduce these "new" topics in the high schools was based on the opinion of educators and/or mathematicians. Research to date has given little definitive information as to their value.

Research workers have been active in studying the effects of programed instruction on the learning of mathematics. According to their results, students do learn through programed instruction. The same is true of TV instruction; however, research indicates that neither is the answer to instruction that is both more effective and more economical.

The teaching demands of college staffs discourage educational experimentation and the study of the college curriculums. Some

teachers are making studies, preparing innovative material, and trying new methods of instruction.

In order to update elementary and high school teachers, college professors have developed inservice material. Instruction, both for teachers and for undergraduate students, has been given by TV, programed textbooks, large lecture sections, and by graduate assistants. Research shows little advantage in any one method.

The search for criteria to predict success in further study of mathematics continues to intrigue investigators. Research shows that teachers' marks are not accurate measures of subject matter achievement. However, grades earned in high school mathematics were better indicators of success in college mathematics courses than IQ scores or scores from a mathematics placement test.

The Educational Research Information Center (ERIC) of the Office of Education anticipates that a mathematics center will soon be established from which significant research and development findings will be available. The center will focus on materials not readily available from commercial publishers. Each item will be prepared for purchase at a nominal sum in both microfiche copy and in pamphlet form.

SUPPORT FOR IMPROVING MATHEMATICS EDUCATION

A knowledge of mathematics is so vital to scientific and technological advancements that intensive and long-range support for improvements in mathematics education is imperative. Efforts include sweeping reform in the mathematics curriculum toward a conceptual approach with some adaptations to accommodate the trend toward use of computers, widespread attention to preservice and inservice teacher education programs to develop skills in using the new methods and materials, and assistance to schools in selecting and purchasing appropriate instructional equipment and materials.

Just as advancements in mathematics are intimately related to advancements in science, so the support for improvements in mathematics is frequently listed with support for science in such a way that it becomes impossible to disentangle the two. With the trend toward coordination and combination of support from

public and private sources, the headings below do not represent mutually exclusive programs.

The Federal Government

National Aeronautics and Space Administration.— Commonly referred to as NASA, this agency provides funds for a wide variety of activities in the space sciences, including mathematics. In 1966, in addition to active research grants and contracts for 29 specific mathematics studies in the physical science area, the agency is supporting such activities as development of space-related supplementary materials for inservice mathematics instruction for elementary school teachers.

National Science Foundation.—An important vehicle of the Federal Government in providing support and encouragement for the development of excellence in mathematics has been the National Science Foundation, working through the mathematical community of scholars in the institutions of higher learning. Since its establishment in 1950, the Foundation has invested almost $200 million in the improvement of mathematics education in the schools and colleges and over $60 million in support of basic research in mathematics by faculty members and graduate students in American universities.

The breadth and extent of this support given to colleges and universities to conduct projects in aid of mathematics education is shown in the outline of major program areas for grants made during the year ending June 30, 1966.

At the university level, the National Science Foundation supports the basic research activities of the graduate faculty, including the research training of their graduate students, through grants in partial support of advanced research in mathematics. In the year ending June 30, 1966, the Foundation made 380 grants, totaling about $12,500,000, for this purpose. At the same time, grants totaling $9 million were made in partial support of university computing facilities, to be used both by mathematicians and by scientists in other fields.

U.S. Office of Education.—Several programs administered by the Office of Education provide support for improvement of mathematics education in the Nation's schools and colleges.

NATIONAL FOUNDATION SUPPORT FOR MATHEMATICS EDUCATION, 1965–1966

Program	Number of projects	Total grants ($ million)	Participants in projects		
			Secondary students	Secondary teachers	Elementary teachers
ELEMENTARY AND SECONDARY SCHOOLS					
Summer institutes for teachers	214	8.744	....	7,385	533
Academic year institutes (full-time and part-time)	200	5.718	....	7,371	2,688
Cooperative college-school training	23	.887	732	754	2,103
Summer training for high-ability students	49	.513	2,326	....	....
Course content improvement	11	2.774	....	....	....
Other training projects	7	.084	....	70	....
	504	18.720	3,058	15,580	5,324
			Participants in projects		
			College students	Graduate students	Mathematics faculty
COLLEGES AND UNIVERSITIES					
Institutes for teachers	15	.823	....	....	416
Advanced seminars	5	.303	....	80	148
Undergraduate research participation	34	.437	396	....	....
Undergraduate instructional equipment	38	.507	....	....	....
Curriculum improvement	8	1.343	....	....	....
Fellowships for graduate or advanced study	....	7.177	....	1,162	132
Special and developmental projects	7	1.248	....	....	....
	107	11.838	396	1,242	696

Total number of projects: 611.
Total amount of grants (estimated): $30,558,000.
Total number of individual participants benefited: 26,296.

As a result of 1958 legislation, the Office is authorized to strengthen instruction in mathematics by means of grants to the States on a matching basis for the acquisition of specialized equipment and materials and for the employment of State specialist supervisors and related service personnel.

During the first seven years, the States received matching funds for over 96,000 approved projects, involving specialized mathematics equipment and materials costing about $67 million. Purchases ran the gamut from numberline aids to electronic computers and included a variety of multisensory and manipulative aids, calculating devices, and audiovisual equipment. Before the enactment of this legislation, the States employed a total of 14 State specialist supervisors in mathematics or as combination mathematics and science specialists. By 1966, the number had increased to 94 State specialist and consultative personnel with responsibilities in mathematics. These individuals provide orientation of staff to updated course content and methodology and consultation services related to the purchase and use of improved mathematics equipment and materials.

Through 1965 legislation, Federal support is provided to improve the qualifications of experienced teachers and other school personnel through advanced study and to train other college graduates for careers in education. Of the experienced teacher fellowship programs established in colleges and universities for the fall of 1966, three are specifically focused on mathematics or a combination of science and mathematics.

Fellowships for advanced study to attract individuals to college teaching have been supported since 1958. During the first 7 years of this program, 618 of these graduate fellowships were awarded in mathematics and statistics.

Of the research training programs authorized by 1965 legislation to expand the Nation's educational research capabilities, at least two programs will be focused on building competencies for research and development in mathematics.

Through authorizations for support of research conducted outside the Office, funds are used for a variety of individual projects for improvement of mathematics at all levels and in all areas. In some cases, this support is used for the research and development component of operating programs, and, in some cases, it is combined

with support from other agencies and organizations to provide intensive and continuous research, development, and demonstration programs.

Professional Organizations

American Association for the Advancement of Science (AAAS).—As the largest professional organization for science in the United States today, with a membership of over 100,000, the American Association for the Advancement of Science is also the parent body of many other professional science and mathematics groups. Maintaining a secretary and also a standing committee for science and mathematics teachings, the AAAS sponsors (1) nationwide programs to improve science and mathematics teaching and to inform school administrators where to get information about new curriculums, up-to-date course materials, and effective teaching aids; (2) cooperative programs with State departments of education to raise certification standards for science and mathematics teachers; and (3) experimental programs to improve the content of science and mathematics courses.

National Academy of Sciences-National Research Council (NAS–NRC).—A nongovernmental agency established as an advisory body on science and technology to the U.S. Government, the Academy furthers a wide variety of research and educational activities in pure and applied sciences, including mathematics.

The Committee on Support of Research in the Mathematical Sciences is currently making a review of the level and form of support of research in mathematical sciences in the United States and also of the undergraduate and graduate levels of education required to carry out this research. The study is concerned with the application of mathematics to all fields as well as with research in mathematics itself.

The Academy Research Council has just issued (March 1966) a report entitled "Digital Computer Needs in Universities and Colleges," a study of educational activities at the undergraduate and graduate level which involve use of modern computing machines.

National Council of Teachers of Mathematics (NCTM).—Affiliated with the American Association for the Advancement of Science, the Conference Board of the Mathematical Sciences, and the

National Education Association, the NCTM serves as an agency for both the coordination and the stimulation of major efforts to improve the teaching of mathematics. The organization publishes three journals—one addressed to secondary school mathematics teachers, one for elementary school mathematics teachers, and one containing enrichment and recreational mathematics materials for students. Besides yearbooks on timely problems in the teaching of mathematics, it also issues booklets and pamphlets on a wide range of topics of interest to mathematics teachers.

Other examples of the many and varied activities of the NCTM include production of films for the inservice education of elementary school teachers; studies of ways to improve mathematics instruction through better use of educational media; preparation of instructional materials for non-college-bound students; and assistance to area, State, and local affiliates to help them be more effective in stimulating professional growth among mathematics teachers.

DISSEMINATION OF INFORMATION ABOUT MATHEMATICS

Before new mathematics programs could be introduced successfully into the school systems, it was necessary that administrators (superintendents and principals) and central instructional staff (supervisors and assisting teachers) were informed about new programs and convinced of their worth. In order to do this, regional conferences were held across the country. These provided opportunities for participants to become informed and to share ways and means of making improvements in their individual programs. State departments of education also held similar regional conferences within the States.

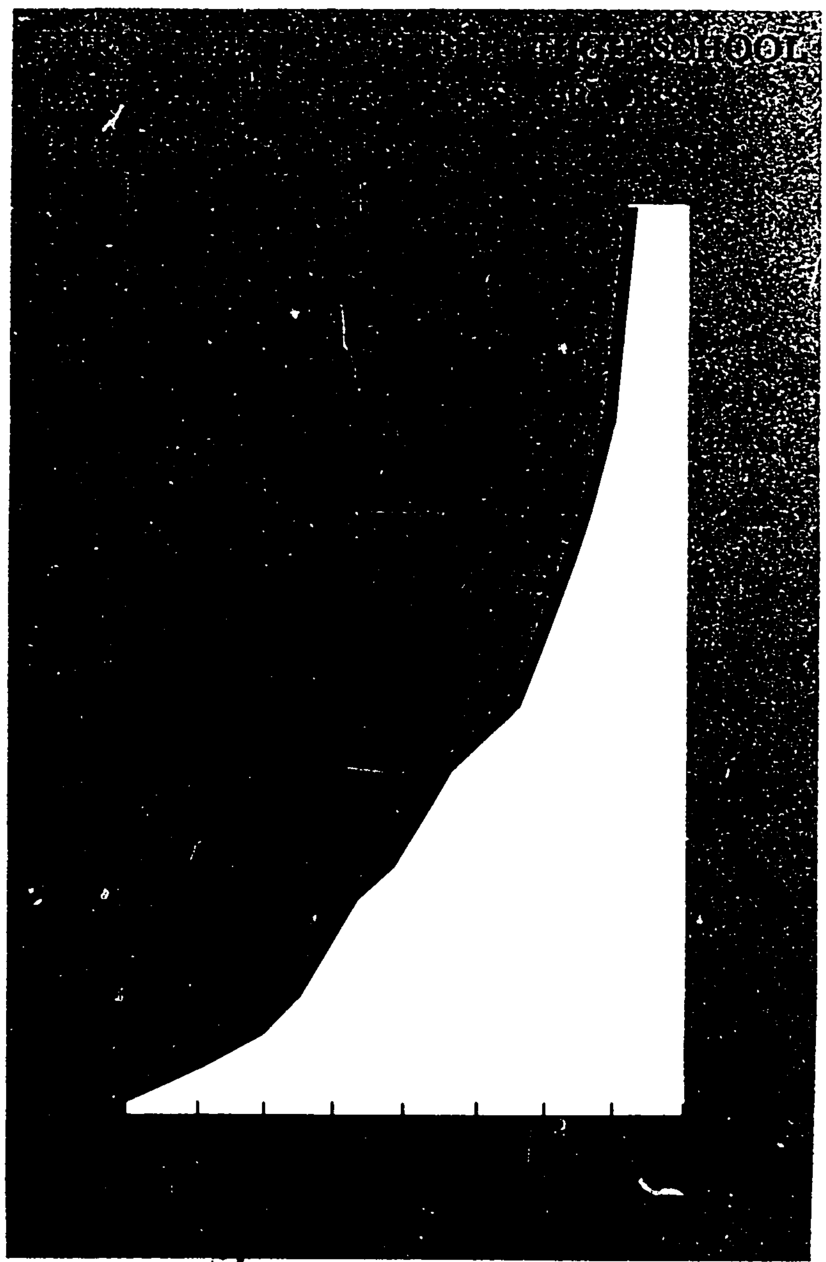
Local school districts or school systems extended these opportunities to the cities and counties. The various professional organizations highlighted new mathematics programs at their meetings and in their journals in a concentrated effort to reach all persons in leadership positions. The National Council of Teachers of Mathematics prepared a film, "Mathematics for Tomorrow," which has been widely circulated in all sections of the Nation.

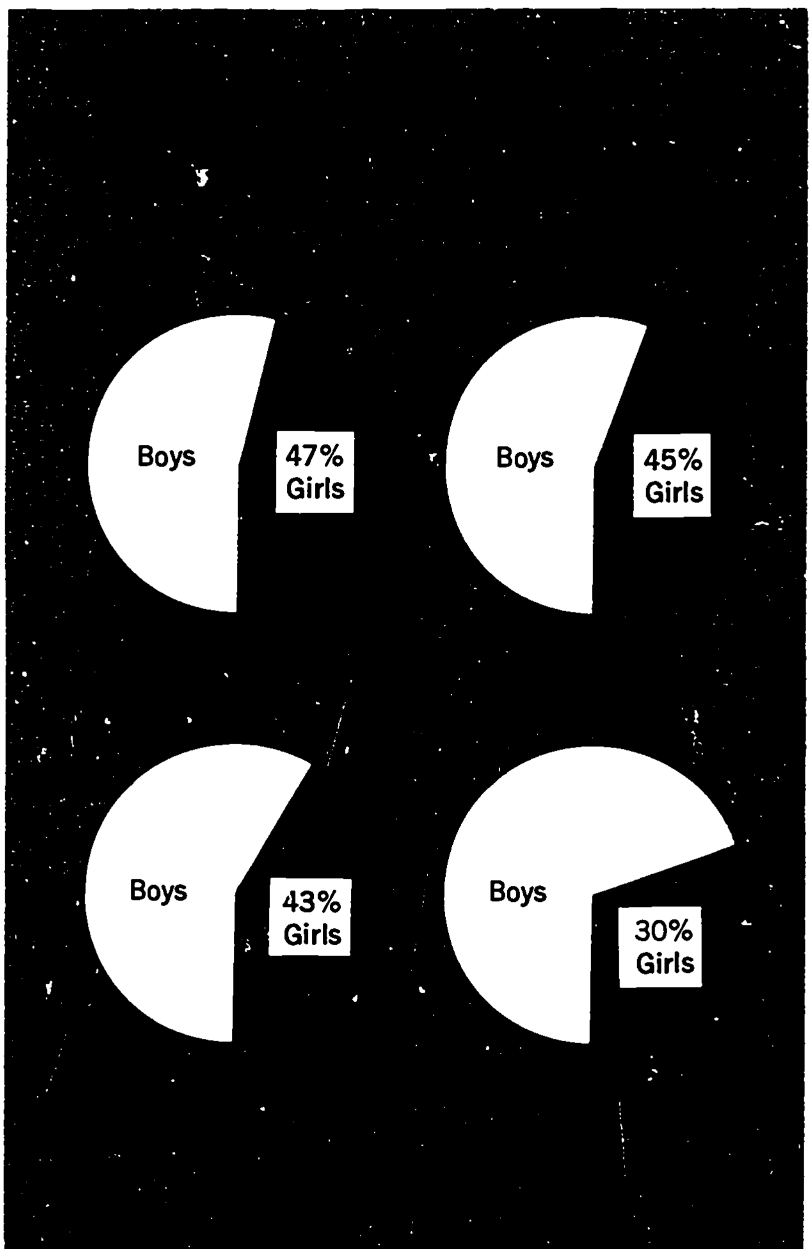
State Departments of Education and local school systems prepared bulletins and leaflets to provide information. Courses for parents

in the new mathematics are being offered. As an example of the great interest of parents, one city found it necessary to hold more than 30 such courses in a single year, with around 50 participants in each. Educational and commercial TV stations have presented series and single programs designed both to inform the public about new mathematical developments and to give parents some insight into the content of these programs. Publishers have prepared textbooks for parent study, and many articles on the new trends in mathematics education have appeared in popular journals.

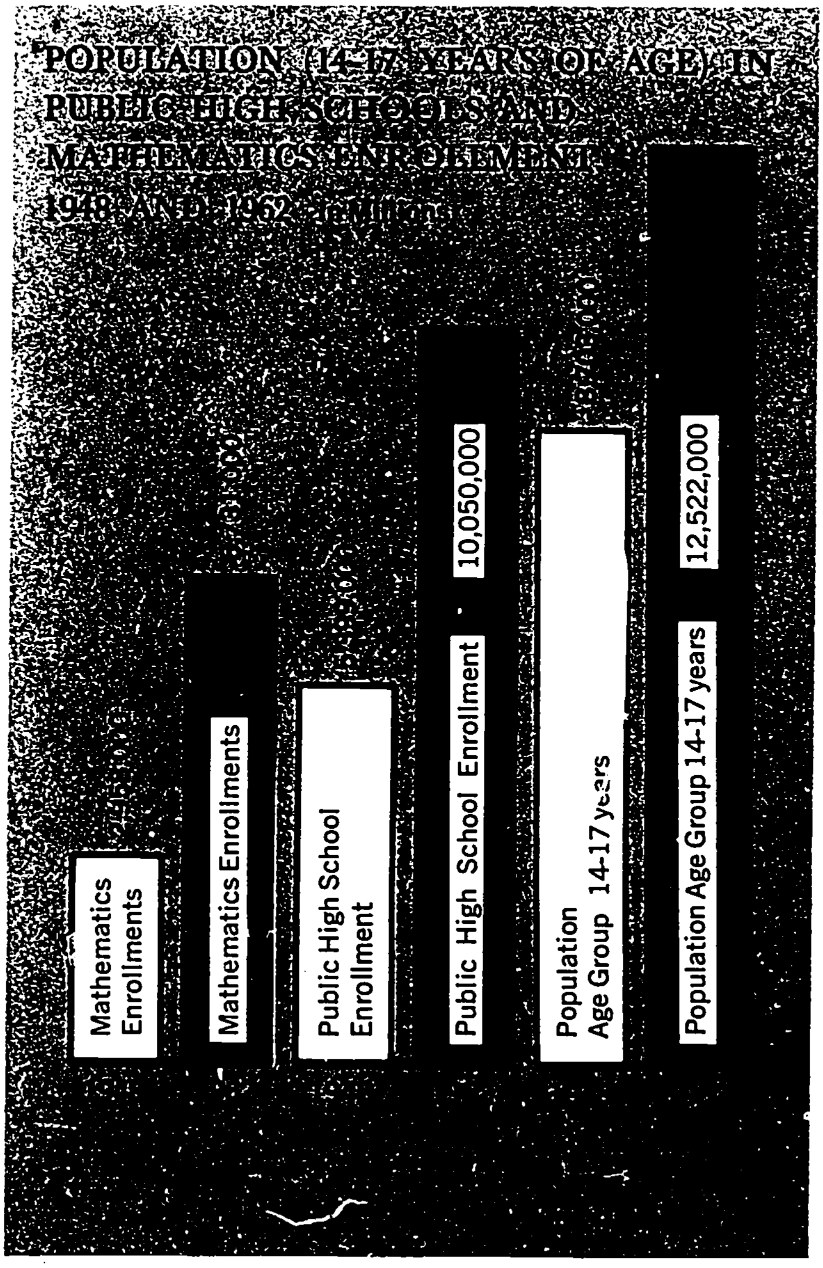
New mathematics programs have aroused interest and curiosity and have stimulated numerous questions among laymen and parent groups. Where mathematics in the elementary and secondary schools formerly received rather placid acceptance, there is now a desire to know what changes are occurring, why they are needed, and toward what ends they are directed.

Independent Exploration





Learning New Symbolism



Symbols to Answer

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SELECTED MATHEMATICS PROJECTS

The Madison Project of Syracuse University and Webster College
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College Park, Maryland.

FACTS ABOUT OFFERINGS AND ENROLLMENTS IN HIGH SCHOOL MATHEMATICS

Total enrollments in mathematics rose 128 percent from 1948–49 to 1962–63, with a new high total of 6,731,000.

In 1962–63 about two-thirds of the public high schools indicated that they were using or expected to use the "new" mathematics material.

Between 1948–49 and 1962–63, the enrollments in plane geometry increased 143 percent and in intermediate algebra 173 percent.

Although the population age group increased 44 percent between 1948 and 1962 and the total public high school enrollments 86 percent, the mathematics enrollments increased 128 percent.

U.S. GOVERNMENT PRINTING OFFICE: 1955 O—217-514